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# A Teaching Practicum in Mathematics

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# A TEACHING PRACTICUM IN MATHEMATICS

An Interactive Qualifying Project Report  
submitted to the Faculty of  
WORCESTER POLYTECHNIC INSTITUTE  
in partial fulfillment of the requirements for the  
Degree of Bachelor of Science  
by  
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Approved:  
Professor John A. Goulet, Major Advisor

# Abstract

In the fall of 2011, I was a student teacher at North High School in Worcester, Massachusetts. I taught one honors-level Precalculus course and two honors-level Algebra II courses. My primary goal for this project was to successfully teach mathematics in a manner that accounted for the diverse range of students. When planning how to conduct classes and preparing materials, I took into account the varying learning styles, learning disabilities, and behavioral issues present in the classroom. Teaching was quite humbling- I now hold my teachers in an even higher regard for overcoming the daily struggles in a classroom. I am grateful to have had the pleasure of helping to make a difference in the academic lives of the students I taught, and overall the experience was immensely enriching.

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# Chapter One

North High School is a public four-year high school located in Worcester, Massachusetts. In the 2010-11 academic year, 1149 students were enrolled at North High. (All data is from the 2010-11 academic year, unless otherwise noted.)

The school boasts an ethnically diverse student body, with 43.2% Hispanics, 27.8% Whites, 19% African Americans, 8.7% Asians, 1.2% Multi-Race/Non-Hispanics, 0.1% Native Americans, and 0.1% Native Hawaiians/Pacific Islanders. English is not the first language for about 50% of the students. The gender ratio is relatively equal, with 53.9% Male students and 46.1% Female students.

Set in Worcester, Massachusetts, North High tends towards an inner-city type school profile. Low-income students account for 77.1% of the student population. The graduation rate in 2009-10 was 58.6%. About 81% of graduates in 2009-10 planned on attending either a 2- or 4-year public or private college. While the school has only a fair graduation rate, the teachers at North High are highly qualified, with 98% being licensed in teaching assignment. The student/teacher ratio is about 12 to 1.

North High School's curriculum is based on the Massachusetts Curriculum Framework, a result of the Massachusetts Education Reform Act of 1993 (MERA). The reform instituted sweeping changes in public education over a seven year period, including greater funding, accountability of both students and teachers, and statewide standards for education. MERA calls for statewide curriculum frameworks and standards of learning in all core academic subjects. The frameworks outline specific guidelines intended to be used

both by teachers and school districts in planning the curriculum. Current core academic subjects include the arts, English language arts, foreign languages, comprehensive health, mathematics, history & social science, and science & technology/engineering. As a capstone, a statewide test called the Massachusetts Comprehensive Assessment System (MCAS) was created to assess the academic standards outlined in the curriculum frameworks. As mandated by MERA, all students must pass the tenth-grade MCAS (as well as meeting local requirements) to receive a diploma. The Act also raises expectations for teachers: since 1998, all teachers are required to pass a test in knowledge of subject content as well as a test in communication and literacy skills to teach in Massachusetts public schools.

In terms of academic performance and assessment, North High reports poor to fair scores on standardized tests. The Massachusetts Comprehensive Assessment System, or MCAS, is a statewide test administered in the tenth grade to all public school students. Students must pass the test to be eligible to receive a high school diploma. It has three components: English Language Arts, Mathematics, and Science & Technology Engineering. For the spring 2011 MCAS, North High reported 61% of students scoring “Proficient or Higher” on English Language Arts, 48% of students scoring “Proficient or Higher” on Mathematics, and only 28% of students scoring “Proficient or Higher” on Science & Technology Engineering (see table).

## MCAS results for North High:

### MCAS Tests of Spring 2011

#### Percent of Students at Each Performance Level for North High



\* NOTE: Performance level percentages are not calculated if student group less than 10.

\* NOTE: Grade 10 Science and Technology/Engineering results represent the highest performance level attained by class of 2013 students in grades 9 or 10 in any of the four subjects (Biology, Chemistry, Introductory Physics, and Technology/Engineering). In addition, only students enrolled in Massachusetts since October 2009 are included in state-level results; only students enrolled in the same district since October 2009 are included in district-level results; only students enrolled in the same school since October 2009 are included in school-level results.

Data Last Updated on September 30, 2011.

Grade and Subject	Proficient or Higher		Advanced		Proficient		Needs Improvement		Warning/ Failing		Students Included	CPI	SGP	Included in SGP
	SCHOOL	STATE	SCHOOL	STATE	SCHOOL	STATE	SCHOOL	STATE	SCHOOL	STATE				
GRADE 08 - ENGLISH LANGUAGE ARTS		79		20		59		15		6	2		N/A	N/A
GRADE 08 - MATHEMATICS		52		23		29		27		21	1		N/A	N/A
GRADE 08 - SCIENCE AND TECH/ENG		39		4		35		42		19	1		N/A	N/A
GRADE 10 - ENGLISH LANGUAGE ARTS	61	84	12	33	49	51	26	13	13	3	211	82.0	46.0	128
GRADE 10 - MATHEMATICS	48	77	26	48	22	29	26	16	25	7	209	70.8	56.0	129
GRADE 10 - SCIENCE AND TECH/ENG	28	67	4	20	24	47	52	27	20	7	166	66.4	N/A	N/A
ALL GRADES - ENGLISH LANGUAGE ARTS	62	69	12	17	50	52	25	23	13	8	213	82.2	46.0	128
ALL GRADES - MATHEMATICS	48	58	26	24	22	34	26	27	26	15	210	70.6	56.0	129

(Source:

<http://math.worcesterschools.org/modules/cms/pages.phtml?pageid=120057&sessionid=075c3c46f0f8e5336eefc3260a8c587f#math>)

# Chapter Two

Worcester Public High Schools follow the “Traditional Model Pathway” for mathematics as seen in the Curriculum Frameworks. Generally speaking, students take an Algebra I course during the ninth grade, a Geometry course during the tenth grade, and an Algebra II course during the eleventh grade. While students are only required to take these three courses, more advanced courses such as Precalculus, Advanced Placement Statistics, and Advanced Placement Calculus are offered. However, students must take Algebra II before enrolling in Precalculus or AP Statistics, and Precalculus before enrolling in AP Calculus. Though the structure is rigid for those students who struggle to pass any one of Algebra I, Geometry, or Algebra II, it also allows more advanced students to further continue their mathematical education with enriching advanced courses.

During my semester-long teaching practicum at North High School, one of the courses I taught was an honors-level Algebra II. The course objectives are as follows (as stated in the Worcester Public Schools High School Curriculum Syllabus): to solve problems using systems of linear equations; to use matrices to solve problems; to solve quadratic equations with complex roots; to utilize the inverse relationship between exponential and logarithmic functions; to use polynomial functions in the solution of problems; and to solve trigonometric equations.

**Worcester Public Schools  
High School Curriculum**

**Course Syllabus– Part I**

**Course Title:** Advanced Algebra

**Course Description:**

The course will focus on the Algebra II Massachusetts Mathematics Curriculum Framework and the Worcester Public Schools 11<sup>th</sup> Grade Mathematics Curriculum. This course is a bridge from Algebra I into advanced topics in mathematics. This is the prerequisite to Pre-calculus and Advanced Placement Statistics.

**Course Objectives:**

Students will:

- Solve problems using systems of linear equations
- Use matrices to solve problems
- Solve quadratic functions with complex roots
- Utilize the inverse relationship between exponential and logarithmic functions
- Use polynomial functions in the solution of problems
- Solve trigonometric functions

**Essential Questions:**

1. How are non-linear situations represented in mathematics?
2. Where do trigonometric functions and solutions occur in our world?

**Contextual Vocabulary:**

polynomial  
exponential  
logarithmic  
matrix  
trigonometric  
combinatorics

**Prerequisite Courses:**

Algebra I and Geometry

(For full syllabus, see Appendix A.) Algebra I and Geometry are prerequisites for Algebra II. It is essential that certain topics are exhausted in these previous courses for a student to be successful in Algebra II.

Essential to a student's success in Algebra II are several topics in Algebra I, including algebra, functions, statistics and probability. Within algebra, a student should be able to:

- perform arithmetic operations on polynomials (A-APR 1 of the March 2011 Curriculum Framework for Mathematics)
- create equations that describe relationships [between dependent and independent variables] (A-CED 1-4)
- solve equations in one variable (A-REI 3-4)
- solve systems of equations (A-REI 5-7)
- represent and solve equations graphically (A-REI 10-12).

For example, a student must know how to perform arithmetic operations on polynomials before they can solve systems of linear equations by an elimination method, as taught in Algebra II. Within functions, a student should be able to:

- build a function that models a relationship between two quantities (F-BF 1-2)
- build new functions from existing functions (F-BF 3-4)
- construct and compare linear, quadratic, and exponential models and solve problems (F-LE 1-3).

For instance, if a student did not know how to solve a quadratic equation with real roots, then they certainly would struggle to solve quadratic equations with complex roots. Within statistics and probability, students should be able to interpret linear models (S-ID 7-9).

Thorough coverage of specific topics in Geometry is also essential in order for students to be successful in Algebra II. Heavy emphasis should be placed on right triangles and trigonometry, as it is crucial for students to be comfortable with these ideas before they can solve trigonometric equations. Specifically, after completing a Geometry course, students should be able to:

- define trigonometric ratios and solve problems involving right triangles (G-SRT 6-8) and
- apply trigonometry to general triangles (G-SRT 9-11).

The other course I taught while at North High was honors-level Precalculus. The course serves as a further investigation into some topics covered in Algebra II, as well as a precursor to Advanced Placement Calculus. The course objectives are as follows (as stated in the Worcester Public Schools High School Curriculum Syllabus): to solve problems using polynomial, power, and rational functions; to solve exponential, logistic, and logarithmic equations; to investigate trigonometric functions and analytic trigonometry; to utilize vectors, parametric equations, and polar equations to solve problems; to expand analytic geometry from two to three dimensions; and to explore calculus including limits, derivatives, and integrals. While some topics in Precalculus parallel those in Algebra II, the Precalculus course delves deeper into the material and emphasizes application. The course

introduces students to some topics covered in Calculus as well as provides a strong foundation to transition into Calculus.



**Worcester Public Schools  
High School Curriculum**

**Course Syllabus – Part I**

**Course Title:** Pre-Calculus

**Course Description:**

The course will focus on the Pre-Calculus Massachusetts Mathematics Curriculum Framework and the Worcester Public Schools 11<sup>th</sup> and 12<sup>th</sup> Grade Mathematics Curriculum. Pre-Calculus is the prerequisite to AP Calculus or a college level calculus course.

**Course Objectives:**

Students will:

- Solve problems using polynomial, power and rational functions.
- Solve exponential, logistic, and logarithmic functions.
- Investigate trigonometric functions and analytic trigonometry.
- Utilize vectors, parametric equations, and polar equations to solve problems.
- Expand analytic geometry from two to three dimensions.
- Explore calculus including limits, derivatives, and integrals.

**Essential Questions:**

1. How are vectors and polar equations used to describe our world?
2. How can calculus be used as an elegant solution to common problems?

**Contextual Vocabulary:**

vectors  
parametric equations  
polar equations  
multivariate systems  
limits  
derivatives  
integrals

**Prerequisite Courses:**

Successful completion of Algebra II

(For full syllabus, see Appendix A.) The prerequisite for the course is successful completion of Algebra II. Since I have already outlined which elements from Algebra I and Geometry must be exhausted for a student to be successful in Algebra II, I will only outline which topics in Algebra II must be exhausted for a student to be successful in Precalculus.

Areas of study in Algebra II that are crucial to success in Precalculus include topics in number and quantity, algebra, and functions. Within number and quantity, students should be able to:

- perform arithmetic operations with complex numbers (N-CN 1-2)
- use complex numbers in polynomial identities and equations (N-CN 7-9) and
- represent and model vector quantities (N-VM 1, 3).

For example, a student must first know what a vector represents in order to use them to solve problems. Critical standards to be covered in algebra include:

- understanding the relationship between zeros and factors of polynomials (A-APR 2-3)
- using polynomial identities to solve problems (A-APR 4-5) and
- representing and solving equations graphically (A-REI 11).

In order for students to solve problems using polynomial functions, they must first be familiar with properties and identities of such functions. Topics in functions to be covered in Algebra II that are vital to success in Precalculus include:

- interpreting functions that arise in applications in terms of content (F-IF 4-6)
- analyzing functions using different representations (F-IF 7-9)

- building functions (F-BF 1, 3-4)
- extending the domain of trigonometric functions using the unit circle (F-TF 1-2)
- modeling periodic phenomena with trigonometric functions (F-TF 5) and
- proving and applying trigonometric identities (F-TF 8).

A strong background in trigonometry (gained in Algebra II) helps ensure a student's success in more advanced and complex trigonometric applications to be covered in Precalculus.

# Chapter Three

There were several factors that went into the preparation and generation of materials I used to both enrich and assess the students' learning experience. Such materials include daily agendas, homework assignments, worksheets, and formal assessments.

An important part of how I structured the courses I taught while at North High was creating a daily agenda. Written on the whiteboard every day, the agenda told students what would be happening in class that day. They included a run-through of the period as well as objectives and homework assignments. I would often begin the class with questions on the previous night's homework or a warm-up exercise. Here is an example of an agenda for Precalculus on September 30<sup>th</sup>, 2011:

## Honors Precalculus

09/30/11

### Agenda:

1. Go over yesterday's quiz on 2.1-2.3
2. Questions on homework
3. Objectives:
  - Fundamental theorem of algebra
  - Complex conjugate pairs
4. Homework: pg. 140-141 #s: 5-8, 10, 13, 16, 18, 20, 28, 30, 36, 38, 46, 50, 52, 58, 60, 64, 68, 72, 76

Agendas for Algebra II are structured the same way. For the collection of all daily agendas, please see Appendix B. The purpose of the agenda is not only to inform the students, but also to act as an informal record of the class' progress.

When creating homework assignments, my aim was to expose the students to several different approaches and types of problems. A single assignment covering a specific topic would usually include vocabulary questions, explicitly computational problems (whether open-ended or multiple choice), graphical approaches, theoretical questions, and applications or word problems. The goal of the homework was to help students further develop and perfect skills learned in class, as well as practice practical applications of these skills. Homework assignments were also designed as a preparation tool: some specific problems focused on skills that were necessary and essential to future material. While most students considered the homework assignments rather extensive, handing in homework was not required, nor was it graded. In other words, homework assignments simply consisted of suggested problems for students to complete. The beginning ten minutes or so of each class was devoted to answering students' questions about the previous night's homework. Another goal homework assignments aimed to accomplish was that of preparing students for formal assessments (tests or quizzes). Homework was most often selected problems assigned from the class textbook, or sometimes a review worksheet or sample exam in preparation for a test or quiz.

As a supplement to the homework assigned from the textbook, I would often create worksheets to be done in class as a means of developing and practicing learned skills. These worksheets were usually practice exams or review sheets to be completed in class the day before a test or quiz. They served as a studying tool for students in that they were designed to give students an idea of what to expect on an assessment, as well as what quality of work was demanded. After spending some class time attempting the worksheets, we would review them together as a class. Students were expected to show all work

leading to their solutions. The questions on the worksheets usually paralleled those found in homework or examples done in class. Here is an example of a review worksheet done in Precalculus the day before a chapter test:

# Ch. 2 Review

- ① Write the equation of the quadratic function (in standard form) whose vertex is  $(3, -6)$  and who passes through the point  $(0, 3)$ .

$$y = a(x-3)^2 - 6$$

$$3 = a(0-3)^2 - 6$$

$$3 = 9a - 6$$

$$9 = 9a$$

$$a = 1$$

$$\boxed{y = (x-3)^2 - 6}$$

- ② Use long division to divide  $3x^3 + 4x - 1$  by  $x^2 + 1$ .

$$\begin{array}{r} 3x + \frac{x-1}{x^2+1} \\ 3x^3 + 0x^2 + 4x - 1 \\ - (3x^3 + 0x^2 + 3x + 3) \\ \hline 3x - 4 \end{array}$$

$$\boxed{3x + \frac{x-1}{x^2+1}}$$

- ③ Find the vertex and x-intercepts (zeros) of  $f(x) = x^2 + 4x + 3$ . (hint: to find vertex, complete the square and put in standard form!)

$$\text{x-int: } f(x) = (x+3)(x+1)$$

$$x = -3 \quad x = -1$$

$$\boxed{(-1, 0) \text{ and } (-3, 0)}$$

$$(x^2 + 4x + 4) + 3 - 4$$

$$= (x+2)^2 - 1$$

$$\boxed{\text{vertex} = (-2, -1)}$$

- ④ Use synthetic division to evaluate  $f(-2)$  for  $f(x) = 3x^4 - 6x^2 + 5x - 1$  (hint: remainder theorem!)

$$\begin{array}{r|rrrrr} -2 & 3 & 0 & -6 & 5 & -1 \\ & \downarrow & -6 & 12 & -12 & 14 \\ \hline & 3 & -6 & 6 & -7 & 13 \end{array}$$

$$\boxed{13}$$

For more examples of created worksheets in Precalculus, please see Appendix D, part I.

Perhaps the most direct indicator of comprehension that I utilized during my student teaching was formal assessment. Formal assessments included quizzes (usually covering two or three sections of a chapter) and tests (covering the entire chapter). The goal of these assessments is to evaluate the students' proficiency and comprehension of the subject by posing various types of questions and applications. Questions that I generated and selected were distributed primarily among three categories: computational, graphical, and applications/word problems. Computational problems involved applying specific mathematical formulas or techniques to an explicit problem. Graphical problems required that students use a graphical approach to solve a problem. Applications and word problems assessed the ability of students to recognize and apply learned techniques to solve a real-life problem. These questions often paralleled in-class examples, problems from homework assignments, and problems on review worksheets. Below is an example of a quiz I created for Algebra II:



Name: KEY

Period 5

# Quiz 10 possible points

For 1 and 2, use the given matrices to evaluate the expression.

$$A = \begin{bmatrix} 1 & -4 \\ 5 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -6 & -1 \\ 2 & 4 \end{bmatrix}$$

①  $3A + B$  (2 pts)

$$= \begin{bmatrix} 3 & -12 \\ 15 & 6 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -15 \\ 15 & 7 \end{bmatrix}$$

②  $C(B-A)$  (2 pts)

$$= C \left( \begin{bmatrix} 1 & 1 \\ -5 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 1 \\ -5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -5 \\ -18 & -2 \end{bmatrix}$$

③ Solve using Cramer's Rule. (3 pts)

$$6x - 2y = -10$$

$$-3x + 5y = 16$$

$$A = \begin{bmatrix} 6 & -2 \\ -3 & 5 \end{bmatrix}$$

$$\det A = 30 - 6 = 24$$

$$x = \frac{\begin{vmatrix} -10 & -2 \\ 16 & 5 \end{vmatrix}}{24} = \frac{-50 + 32}{24} = \frac{-18}{24} = -\frac{3}{4}$$

$$y = \frac{\begin{vmatrix} 6 & -10 \\ -3 & 16 \end{vmatrix}}{24} = \frac{96 - 48}{24} = \frac{48}{24} = 2$$

$$(-\frac{3}{4}, 2)$$

④ The vertices of a sail for a sailboat are given by (0,2), (12,2) and (12,26), measured in feet. Find the area of the sail. (3 pts)

$$\pm \frac{1}{2} \begin{vmatrix} 0 & 2 & 1 \\ 12 & 2 & 1 \\ 12 & 26 & 1 \end{vmatrix} = \frac{312 - 24}{356}$$

$$= \pm \frac{1}{2} [(0+24+312) - (24+0+24)]$$

$$= \pm \frac{1}{2} (336 - 48) = \pm \frac{1}{2} (288) = 144 \text{ ft}^2$$

For the collection of all self-generated formal assessments for Precalculus and Algebra II, please see Appendix E.

These formal assessments also served as a tool in evaluating my proficiency as a teacher. By analyzing the students' scores, I was able to draw conclusions about the quality of my teaching. For example, if the majority of the students scored poorly on a particular exam, then it would be safe to say that either the test was too hard or the students did not understand the material well.

The main goal of this collection of materials was to have students cultivate and successfully apply mathematical skills. The materials were designed also to help prepare students for more advanced mathematics by building a strong foundation of mathematical knowledge.

# Chapter Four

During my time at North High, I was exposed to a diverse student population within the classroom. The manner in which I conducted classes was highly dependent on attempting to accommodate all of students' needs in order to successfully teach the material. My aim was to create an environment where all students have the same learning opportunities.

One of the greatest challenges I faced as a teacher was to address the different learning styles, behavioral issues, attendance concerns, and learning disabilities of all members of the class. In creating an outline of how I planned to run the class, I took these factors into account in an effort to accommodate each and every student. The Precalculus course I taught had about fifteen students, and each of the Algebra II courses had about thirty. All classes had about an even ratio of males to females in the class.

Within the classes that I taught, the two prevalent learning styles of students that I encountered were visual learners and auditory learners. The visual learners made up for about two-thirds to three-quarters of the classes (on average), and would tend to prefer pictorial representations of mathematical material. These students would gain the greatest understanding of the material through graphical representations and illustrations. For example, when a visual learner struggled to set up and solve a word problem, I would suggest that they first draw a picture of the problem being described. This would often clarify the problem as well as help them understand what variables are given and which ones they need to solve for. Another specific example of catering to the visual learners was

the derivation and creation of the trigonometric unit circle, which we did together as a class. The activity resulted in a mathematical epiphany for students who had previously not understood where trigonometric properties and values come from (i.e., the 'why' of how trigonometry works).

The auditory learners made up for about one-third to two-thirds of the classes (on average), and tended to prefer aural explanations and instruction. Spoken clarification of the material was the best means of communicating concepts for these students. For the auditory learners, I found that spoken explanations and elaborations were most useful in helping students to understand the material. For instance, when covering new definitions and theorems, the auditory learners often benefitted from extra elaboration. Sometimes simply copying a definition for new mathematical vocabulary was not enough for these students; they needed further elaboration of what the definition meant in terms they could better understand.

Behavioral issues in the classes I taught were only slightly present, and mainly consisted of problems centered on the students' attentiveness. In Precalculus and the period 5 Algebra II classes, attitude issues only affected two or three students. These students were more physically present than mentally- i.e., they were only there because they had to be. In the period 6 Algebra II class, attitude problems were only slightly more prevalent, and were dealt with individually, since it was usually the same students. Distractions for the students included side conversations (at inappropriate times), cell phones, and work for other classes. In the case that a student uses a cell phone in class, I followed the school's policy: the teacher would take it away and either return it after class

or the student can retrieve it in the main office at the end of the day. As for attendance concerns, the greatest problems arose when attendance concerned the class as a whole. Events such as fire drills, grade-wide testing, and school-wide presentations often consumed the entirety or majority of class time. While these events did not occur as often as individual absences did, they were more detrimental to staying on schedule with the material. On any given day, the average number of students who were absent from class was anywhere from one to three. Individual student absences did not affect the prepared agenda and material. Students were made aware of what to expect the next day in class by simply looking ahead in the textbook to the next section. Since I began each class by reviewing and answering questions on the previous nights' homework, students who were absent the previous day had a chance to interactively attempt the problems. I encouraged them to ask questions after class or during a free period (that I was not teaching) if they desired further instruction and review of the material. Learning disabilities generally did not affect the majority of students. For a few students, the ability to complete an assessment in the time allotted was an issue. Usually only a handful (or less) of students struggled to complete an assessment on time.

Taking into account the diversity of the classes I taught, I created an outline of how I would run the class in order to best accommodate all of my students. When teaching new material, I found that a blend of visual representations and verbal elaborations was an excellent means of communicating new concepts. Interactive lectures took up the majority of class time. These consisted of students taking notes on the definitions, theorems, methods, etc. that were to be covered in class that day. New material was always followed with interactive examples. For instance, if students were unsure of how to apply a newly

learned formula, I would make suggestions as to where to start or guide them through the solution. Class participation was strongly encouraged: students would volunteer to write solutions on the white board (so the whole class could follow along). If I felt that a certain student was lacking participation, I would select them to come up to the board and attempt to work through the problem. When a student struggled, other members of the class would offer suggestions and help the student work towards the solution. These interactive examples and problems were an excellent daily indicator of the progress each student was making.

# Chapter Five

In terms of assessments, I utilized both quizzes as well as chapter tests to evaluate the students' knowledge of the subject. Quizzes were typically given once a week, and covered two to three subsections of a chapter. Chapter tests were given when we finished covering an entire chapter in the textbook.

The textbook used for Precalculus was *Precalculus with Limits: A Graphing Approach*, 6<sup>th</sup> edition, by Ron Larson. The book contained several step-by-step examples, and used different colors to highlight important material. Graphical representations and methods were prevalent in the book, and it also gave instructions for how to perform certain actions on a graphing calculator. The teacher's edition of the book contained an abundance of extra examples. I often used these for in-class examples or quiz questions. I thought this textbook was a good, but not perfect, choice for the class- while it included a wide range of applications and methods, it often lacked diversity in computational problems. Having the solutions to the odd-numbered problems in the back of the book was useful for students: since I typically only assigned even-numbered problems, there would usually be a very similar odd-numbered problem that the student can check their answer to.

The quizzes I generated had three or four fairly direct problems. The students were given about fifteen to twenty-five minutes to complete the quiz, based on the material being covered. When creating a quiz, I selected problems systematically based upon examples we had covered in class. The majority of the problems on a quiz were generally similar to those covered in homework assignments or those done during class. I would

often choose to make one of the problems more involved than those we had done in class or on homework assignments, and required a bit more application than the others. For example, if I gave a word problem on a quiz, the methods to obtain the solution would be the same as those we had gone over in class, but perhaps the given variables and the variable they are asked to solve for would be different. In selecting and generating the quiz, I often placed more emphasis on topics that had been exhausted in class.

The chapter tests I generated covered a wide range of material from an entire chapter in the textbook. They usually consisted of anywhere from six to ten problems, based on how time-consuming the material was. Students were given the entire class period (forty-five minutes) to complete the test. About half of the problems on a test would be similar to those found on the quizzes covering sub-sections of that chapter, and the other half would involve applications and/or graphical approaches. For a few of the tests, I added a bonus problem which was a new application or a “challenge” problem that students could attempt for extra credit (no credit was lost if the problem was not attempted).

In Precalculus, by the end of my term as a student teacher, I felt that about half of the students were capable of continuing on to study Calculus. Though certainly more than half of the class had grades that would have indicated they were ready for Calculus, only half put in the effort that would be needed to succeed in a Calculus course.

In all three of the classes I taught, the students were either in the 11<sup>th</sup> or 12<sup>th</sup> grade, so preparing the students for MCAS testing was not applicable (since the last MCAS tests are given in 10<sup>th</sup> grade).



# Conclusion

Overall, my experience as a student teacher at North High was both rewarding and enriching. The practicum provided me with valuable knowledge about the teaching profession, including both the informational and the socio-psychological aspects of it.

While choosing to teach in an 'inner-city' type school was certainly more of a challenge for me than teaching in a suburban school, the choice was undoubtedly the right one. Perhaps the most prevalent reason I had for selecting this practicum as my Interactive Qualifying Project was the desire to share my passion for mathematics with others. One of the goals I kept in mind while teaching at North High was to reach out to students and try to make math more exciting and enticing to them. Though there will always be students who dislike math, I attempted to make it as enjoyable as possible. When students have an interest in the material or a desire to learn more, success in the class usually follows. Having students who were interested in math not only made it more enjoyable for the students, but also more enjoyable for me as their teacher.

Another goal I kept in mind as a student teacher was to create a classroom environment that nurtured both academic success and an overall amicable disposition. Putting my best foot forward every day, I greeted students at the door as they came in to class with a smile. Students were quite receptive to the positive attitude I brought into the classroom. Within a couple weeks of teaching, I knew almost all of my students by name; I believe this is important to help give students a sense of amiability, instead of being just a name on a test.

Some of the most rewarding aspects of the experience happened on a smaller scale within the classroom every day. For instance, sometimes when students had great difficulty applying new methods to solve a problem, I would show them a different way to get to the solution. This usually elicited an “Oh-I-get-it-now!” moment, which brought joy to both the students and me. Another rewarding experience occurred when students would make great progress or improvement. For example, when a student was able raise their overall grade in the class by a letter. It brought a great sense of accomplishment to both the student, for improving their grade, and to me, for successfully communicating the material.

Being a student teacher at North High was a humbling experience. It gave me insight as to how difficult it can be at times to get through to students and successfully manage a classroom. Looking back, I can say that I am deeply grateful for having the opportunity to take part in such an experience. The hands-on aspect of the practicum gave me the chance to make a difference in the lives of students. At the end of my duration as a student teacher, the classes I had taught each gave me a card signed by all the students. Hearing that I had made a difference in the academic careers of my students was incredibly moving. Even though I am no longer their teacher, I feel I have given them the ability and foundations they need to succeed in the future. Perhaps the largest thing I have taken away from this experience is the confidence that I made a difference in the lives of these students.

# Appendix A

## North High Statistics & Syllabi

### 1. North High demographics for 2010-2011:

Enrollment by Race/Ethnicity (2010-11)			
Race	% of School	% of District	% of State
African American	19.0	13.6	8.2
Asian	8.7	8.1	5.5
Hispanic	43.2	38.3	15.4
Native American	0.1	0.3	0.2
White	27.8	36.5	68.0
Native Hawaiian, Pacific Islander	0.1	0.0	0.1
Multi-Race, Non-Hispanic	1.2	3.1	2.4

Enrollment by Gender (2010-11)			
	School	District	State
Male	619	12,471	490,363
Female	530	11,721	465,200
Total	1,149	24,192	955,563

	School	District	State
Attendance Rate	89.4	94.2	94.7
Average # of days absent	17.0	9.8	9.1
In-School Suspension Rate	19.3	7.2	3.5
Out-of-School Suspension Rate	21.0	12.6	5.6
Retention Rate	13.8	3.5	2.1
Graduation Rate	55.1	72.0	83.4
Truancy Rate	46.5	32.0	2.5

Title	% of School	% of District	% of State
First Language not English	52.6	43.8	16.7
Limited English Proficient	24.8	28.6	7.3
Low-income	81.2	72.1	35.2
Special Education	24.4	20.7	17.0
Free Lunch	74.6	65.8	30.4
Reduced Lunch	6.6	6.3	4.8

(Source: <http://profiles.doe.mass.edu/profiles/student.aspx?orgcode=03480515&orgtypecode=6&>)

## 2. Worcester Public Schools Syllabus for Algebra II (Advanced Algebra):

**Worcester Public Schools**

**High School Curriculum**

**Course Syllabus– Part I**

**Course Title:** Advanced Algebra

**Course Description:**

The course will focus on the Algebra II Massachusetts Mathematics Curriculum Framework and the Worcester Public Schools 11<sup>th</sup> Grade Mathematics Curriculum. This course is a bridge from Algebra I into advanced topics in mathematics. This is the prerequisite to Pre-calculus and Advanced Placement Statistics.

**Course Objectives:**

Students will:

- Solve problems using systems of linear equations
- Use matrices to solve problems
- Solve quadratic functions with complex roots
- Utilize the inverse relationship between exponential and logarithmic functions
- Use polynomial functions in the solution of problems
- Solve trigonometric functions

**Essential Questions:**

1. How are non-linear situations represented in mathematics?
2. Where do trigonometric functions and solutions occur in our world?

**Texts:**

Holt, Rinehart, and Winston; Advanced Algebra; 2003.

### District-Wide Reading Skills Across the Curriculum:

- **Preview** (survey) – note major elements such as organization, vocabulary, summary and graphics.
- **Ask Questions** - question the text, the author and self.
- **Activate Prior Knowledge** (schema) – use what is already known to enhance understanding of what is new in the text.
- **Make Connections** - link text to self, text to world and text to text.
- **Visualize** - use sensory images to create a mental picture of the scene, story, situation, or process and involve oneself in it.
- **Draw Inferences** - go beyond the literal information in the text including predicting, figurative meaning and thematic understanding.
- **Distinguish Key Ideas** - recognize main idea and key concepts.
- **Use Fix-Up Strategies** - monitor own understanding by pausing to think, re-read, consider what makes sense, restate in own words.

### Contextual Vocabulary:

Polynomial  
exponential  
logarithmic  
matrix  
trigonometric  
combinatorics

### Recommended Grading Policy (indicate percent for each factor):

- Classroom participation -
- Projects/papers -
- Homework -
- Final test/assessment\* - 10%
- Other \_\_\_\_\_

\*The Worcester School Committee requires that the final test/assessment be 10% of a student's grade

### Prerequisite Courses:

Algebra I and Geometry

**Note to Teachers:** In addition to handing out the above syllabus to students, you should also hand out to them your expectations in the following areas:

- ✓ Homework policy
- ✓ Make-up policy
- ✓ Attendance requirements
- ✓ Any other expectations

**Course Syllabus – Part II, Academic Content for the First Semester**

**Advanced Algebra II**

<b>Content/Topics –</b>	<b>Skills</b>	<b>Required Papers/Projects, Readings, and Final Assessment/Test</b>	<b>Academic Standards (Worcester Benchmarks and State Frameworks)</b>
<b>Linear Representations</b>	<b>Find slope and intercepts</b>		<b>All.P.8 Solve a variety of equations and inequalities using algebraic, graphical, and numerical methods.</b>
<b>Numbers and Functions</b>	<b>Solve linear equations in two variables</b>		
	<b>Use operations with numbers</b>		<b>All.N.2 Simplify numerical expressions with powers and roots. Including fractional and negative exponents.</b>
	<b>Use operations with functions</b>		
	<b>Identify properties of exponents</b>		
<b>Systems of Linear Equations and Inequalities</b>	<b>Solve systems of equations</b>		
	<b>Find solutions to linear inequalities in two variables</b>		
	<b>Solve systems of linear inequalities</b>		<b>All.P.9 Use matrices to solve systems of linear equations.</b>
<b>Matrices</b>	<b>Use matrices to represent data</b>		
	<b>Solve using matrix multiplication</b>		
	<b>Find the inverse of a matrix</b>		<b>All.P.7 Find solutions to quadratic equations and apply to the solutions of</b>

Quadratic functions	<p>Solve quadratic equations</p> <p>Factor quadratic equations</p> <p>Use the completing the square method</p> <p>Solve using the quadratic formula</p> <p>Use complex numbers in the solution to quadratic equations</p>		<p>problems.</p> <p>All.P.4 Demonstrate an understanding of the exponential and logarithmic functions.</p>
Exponential and Logarithmic Functions	<p>Solve problems involving exponential growth and decay</p> <p>Graph and solve exponential functions</p> <p>Graph and solve logarithmic functions</p> <p>Use and apply the properties of logarithms</p> <p>Solve problems using base <math>e</math></p>		



## Course Syllabus – Part II, Academic Content for the Second Semester

## Advanced Algebra II

Content/Topics –	Skills	Required Papers/Projects, Readings, and Final Assessment/Test	Academic Standards (Worcester Benchmarks and State Frameworks)
Polynomial Functions	Graph polynomial functions  Find products and factors of polynomials  Solve polynomial equations  Find the zeros of polynomial functions		All.P.8 Solve a variety of equations and inequalities including polynomial, exponential, and logarithmic functions.  All.P.5 Perform operations on functions, including composition.
Rational & Radical Functions	Identify inverse, joint and combined variation  Graph rational functions  Multiply and divide rational expressions  Add and subtract rational expressions  Solve rational equations and inequalities  Identify radical expressions and functions  Simplify radical expressions		
Conic Sections	Find parabolas, circles, ellipses, and hyperbolas		All.G.3 Relate geometric and algebraic representations of lines, simple curves, and conic sections.  All.D.2 Use combinatorics to solve problems, in particular, to compute

<p><b>Counting Principals</b></p>	<p>Use Permutations and combinations</p> <p>Identify independent events</p> <p>Solve situations involving dependant events and conditional probability</p>		<p>probabilities of compound events.</p>
<p><b>Series and Patterns</b></p>	<p>Solve using arithmetic and geometric sequences</p> <p>Solve using arithmetic and geometric series</p> <p>Use Pascal's triangle in the solution to problems</p> <p>Use the binomial theorem</p>		<p>All.P.2 Identify arithmetic and geometric sequences and finite arithmetic and geometric series.</p>
<p><b>Trigonometric Functions</b></p>	<p>Solve trigonometric functions</p> <p>Find radian measure and arc length</p> <p>Graph trigonometric functions</p> <p>Find inverses of trigonometric functions</p> <p>Use the Laws of Sines and Cosines</p>		<p>All.G.1 Define the sine, cosine, and tangent of an acute angle.</p> <p>All.G.2 Derive and apply basic trigonometric identities and the laws of sines and cosines.</p>

(Source: <http://math.worcesterschools.org/modules/cms/pages.phtml?pageid=120057&sessionid=043bd2107a6339bd137493436c9a78c5>)

### 3. Worcester Public Schools Syllabus for Precalculus:

**Worcester Public Schools**

**High School Curriculum**

**Course Syllabus – Part I**

**Course Title:** Pre-Calculus

**Course Description:**

The course will focus on the Pre-Calculus Massachusetts Mathematics Curriculum Framework and the Worcester Public Schools 11<sup>th</sup> and 12<sup>th</sup> Grade Mathematics Curriculum. Pre-Calculus is the prerequisite to AP Calculus or a college level calculus course.

**Course Objectives:**

Students will:

- Solve problems using polynomial, power and rational functions.
- Solve exponential, logistic, and logarithmic functions.
- Investigate trigonometric functions and analytic trigonometry.
- Utilize vectors, parametric equations, and polar equations to solve problems.
- Expand analytic geometry from two to three dimensions.
- Explore calculus including limits, derivatives, and integrals.

**Essential Questions:**

1. How are vectors and polar equations used to describe our world?
2. How can calculus be used as an elegant solution to common problems?

**Texts:**

Addison, Wesley, and Longman; Pre-Calculus; 2001.

### District-Wide Reading Skills Across the Curriculum:

- **Preview** (survey) – note major elements such as organization, vocabulary, summary, and graphics.
- **Ask Questions** - question the text, the author and self.
- **Activate Prior Knowledge** (schema) – use what is already known to enhance understanding of what is new in the text.
- **Make Connections** - link text to self, text to world and text to text.
- **Visualize** - use sensory images to create a mental picture of the scene, story, situation, or process and involve oneself in it.
- **Draw Inferences** - go beyond the literal information in the text including predicting, figurative meaning and thematic understanding.
- **Distinguish Key Ideas** - recognize main idea and key concepts.
- **Use Fix-Up Strategies** - monitor own understanding by pausing to think, re-read, consider what makes sense, restate in own words.

### Contextual Vocabulary:

vectors

parametric equations

polar equations

multivariate systems

limits

derivatives

integrals

### Recommended Grading Policy (indicate percent for each factor):

- Classroom participation -
- Projects/papers -
- Homework -
- Final test/assessment\* - 10%
- Other \_\_\_\_\_

\*The Worcester School Committee requires that the final test/assessment be 10% of a student's grade

### Prerequisite Courses:

Successful completion of Algebra II

**Note to Teachers:** In addition to handing out the above syllabus to students, you should also hand out to them your expectations in the following areas:

- ✓ Homework policy
- ✓ Make-up policy
- ✓ Attendance requirements
- ✓ Any other expectations

**Course Syllabus – Part II, Academic Content for the First Semester**  
**Pre-Calculus**

Content/Topics –	Skills	Required Papers/Projects, Readings, and Final Assessment/Test	Academic Standards (Worcester Benchmarks and State Frameworks)
Functions and Graphs	Identify functions and their properties Solve and graph the ten basic functions Build functions from functions Transform functions graphically		All.P.8 Solve a variety of equations and inequalities using algebraic, graphical, and numerical methods.
Polynomial, Power and Rational Functions	Model linear and quadratic functions Solve power and polynomial functions Find real zeros of polynomial functions Explore complex numbers Use the Fundamental Theorem of Algebra Solve inequalities in one variable		PC.N.1 Plot complex numbers using the rectangular coordinate system.
Exponential, Logistic, and Logarithmic Functions	Solve exponential and logistic functions Model exponential and logistic functions Graph logarithmic functions Utilize the properties of logarithmic functions		PC.P.7 Translate between geometric, algebraic and parametric representations of curves. Apply to the solution of problems.
Trigonometric Functions	Find trig functions of acute angles Graph trig functions Graph composite trig functions Use inverse trig functions		PC.P.3 Demonstrate an understanding of the trigonometric functions (sine, cosine, tangent, cosecant, secant, and cotangent). Relate the functions to their geometric definitions.
Analytic Trigonometry	Solve problems with trigonometry  Use fundamental identities Prove trigonometric		PC.P.5 Demonstrate an understanding of the formulas for the sine and

	<b>identities</b> <b>Use sum and difference identities</b> <b>Solve problems using multiple-angle identities</b> <b>Use the laws of sines and cosines</b>		<b>cosine of the sum or the difference of two angles.</b>
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**Course Syllabus – Part II, Academic Content for the Second Semester**

**Pre-Calculus**

Content/Topics –	Skills	Required Papers/Projects, Readings, and Final Assessment/Test	Academic Standards (Worcester Benchmarks and State Frameworks)
Vectors, Parametric Equations, and Polar Equations	Find vectors in a plane Solve using dot products of vectors Use parametric equations Solve problems using polar coordinates Graph polar equations Utilize De Moivre's Theorem		PC.G.2 Use the notion of vectors to solve problems. Use vector methods to obtain geometric results. PC.P.5 Relate the formulas to DeMoivre's theorem and use them to prove other trig identities.
Systems and Matrices	Solve systems with two equations Solve using matrix algebra Use multivariate linear systems and row operations Use partial fractions Solve systems of inequalities in two variables		
Analytic Geometry in Two and Three Dimensions	Graph conic sections, parabolas, ellipses, and hyperbolas Utilize translations and rotations of axes Find polar equations of conics Explore the three dimensional Cartesian coordinate system		PC.P.8 Identify and discuss features of conic sections: axes, foci, asymptotes, and tangents.
Introduction to Calculus: Limits, Derivatives, and Integrals	Connect limits to tangent lines Connect limits to area Solve problems using limits Find numerical derivatives and integrals		PC.P.9 Relate the slope of a tangent line at a specific point on a curve to the instantaneous rate of change.

(Source:

<http://math.worcesterschools.org/modules/cms/pages.phtml?pageid=120057&sessionid=043bd2107a6339bd137493436c9a78c5>)

# Appendix B

## Daily Agendas



H. Precalc

09/26

Agenda:

1) ?'s on hw

2) Objectives:

- max/min values of quadratics
- higher-degree polynomials

3) Review Test

4) HW: pp. 109-112 #s: 10-16 even, 26, 28,  
30, 34, 36, 40, 42, 45, 48, 50, 56, 62, 70, 79,  
80, 86, 88, 90, 98, 112, 120.

## H. PreCalc

09/27

### Agenda:

- 1) Review Chapter 1 exam
- 2) Objectives:
  - higher-degree polynomials cont'd.
  - long division of polynomials
- 3) ?'s on homework
- 4) Homework: continue to study 2.2 & 2.3,  
pp. 124-125 #s: 10, 14, 17, 18, 22, 34, 36,  
104-106

## H. PreCalc

09/28

### Agenda

- 1) Objectives
  - Synthetic division
  - Rational zero test
- 2) ?'s on homework
- 3) Homework: pp. 124-126 #s: 26-32 even,  
37, 38, 44, 46, 48, 52-60 even, 84, 86, 88-98  
even, 107-108.

H. Precalc

09/29

Agenda:

1) Quiz on 2.1-2.3

2) Objectives:

- Real zeros of polynomial functions cont'd.
- Complex numbers

3) ? is on homework

4) homework: pp. 125 #s: 61-64, 71-74

pp. 133-134 #s: 12, 15, 16, 18, 22-26 even, 34, 38,  
42, 46, 48, 54-56, 60, 66, 70, 74, 78, 86, 102.

H. Precalc

09/30

Agenda:

1) go over quiz

2) ? is on homework

3) Objectives

1) fundamental theorem of algebra

2) complex conjugate pairs

4) Homework: pp. 140-141 #s 5-8, 10, 13, 16,

18, 20, 28, 30, 36, 38, 46, 50, 52, 58, 60, 64, 68,  
72, 76.

## H. Precalc

10/3

### Agenda:

1) ?'s on homework

2) Objectives

- Rational functions

- asymptotes

3) Homework: pp. 147-150 #s 11-16, 18, 22,  
24, 30, 36, 38, 40, 44, 46, 48, 56

McDougal Littell

Teachers edition

"Algebra 2"

- Larson

- Boswell

- Kanold

- Stiff

$$2x^2 + 7x + 3$$

$$(2x+1)(x+3)$$

#### H. Precalc

10/4

##### Agenda:

- 1) ?'s on homework
- 2) Objectives
  - graphing rational functions
  - slant asymptotes

- 3) Homework: pp. 157-160 #s: 18, 22, 26, 28, 32, 36, 40, 41, 45, 48, 52, 58, 60, 62, 66, 69, 78, 84, 88, 92, 98, 89

#### H. Precalc

10/5

##### Agenda:

- 1) ?'s on homework
- 2) Objectives:
  - Slant asymptotes
  - quadratic models

- 3) Homework: pp. 165-167 #s 4-16 even, 18, 20, 21, 26



#### H. Precalc

10/5

##### Agenda:

- 1) ?'s on homework
- 2) Objectives:
  - slant asymptotes
  - quadratic models
- 3) Homework: pp. 165-167 #s 4-16 even,  
18, 20, 21, 26

#### H. Precalc

10/6

##### Agenda:

- 1) warm-up
- 2) Review for Chapter 2 Test
  - Practice Test
- 3) Homework: STUDY all of Ch. 2,  
pp. 170-174 #s 8, 12, 14, 20, 26, 32,  
34, 38, 42, 50, 56, 60, 63, 66, 68,  
80, 86, 92, 96, 108, 112, 118, 123, 130,  
134, 137, 142, 150, 154, 156, 158, 160.

### H. Precalc

10/11

#### Agenda:

- 1) Chapter 2 Test
- 2) Homework: Look at section 3.1

### H. Precalc

10/12

#### Agenda:

- 1) Review Ch. 2 Test
- 2) objectives:
  - exponential functions
  - graphing exponential functions
- 3) Homework: pp. 189 ~~190~~ #s 12-16,  
17-20, 22, 23, 25, 27, 30, 31, 34,  
36, 43, 48

### H. Precalc

10/14

#### Agenda:

- 1) Objectives:
  - exponential functions
  - the natural base  $e$
  - graphing exponential functions
  - compounded interest
- 2) Homework: pp 189-191 #s 7, 8, 12-16,  
17-20, 22, 25, 30, 31, 34, 36, 43, 48, 50,  
54, 58, 62, 69, 74, 80

## H. Precalc

10/24

### Agenda:

- 1) ?'s on homework
- 2) Objectives:
  - properties of exponential & logarithmic function
  - solving exponential & logarithmic equations
- 4) Homework: pp. 217-220 #s: 19, 20, 22, 28, 32, 36, 41-46, 50, 52, 62, 68, 71, 76, 78, 84, 93, 98, 104, 112, 136, 137, 139, 144, 148, 152, 156
- 3) minute papers

## H. Precalc

10/25

### Agenda:

- 1) Quiz on 3.1-3.4
- 2) ?'s on homework
- 3) ~~Exponential~~ Objectives:
  - Exponential & logarithmic models
- 4) Homework: 228-232 #s 16, 20, 24, 25, 28, 32, 35, 38, 40, 44, 48, 52, 58, 62,



#### H. PreCalc

10/26

##### Agenda:

- 1) Objectives :
  - exponential & logarithmic models
- 2) Return Quiz
- 3) Homework: pp. 228-232 #s 16, 20, 24, 25, 28, 32, 35, 38, 40, 44, 48, 52, 58, 62

#### H. PreCalc

11/2

##### Agenda:

- 1) ?'s on homework
- 2) Chapter 3 Review
- 3) Homework: STUDY FOR Chapter 3 Test,  
pp. 244-247 #s 20, 22, 24, 29, 34, 38, 46,  
50, 56, 62, 68, 76, 89, 90, 98, 104, 112, 116,  
121-126, 128, 134,

#### H. PreCalc

11/3

##### Agenda:

- 1) Chapter 3 Test
- 2) Homework: STUDY 4.1 & come in tomorrow with questions.

$$C = 5\theta \quad \theta = \frac{6}{5} \quad \frac{(2\pi)(1)(14) \text{ inches}}{1 \text{ second}} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{11}{7}$$

# HL Precalc

## Agenda:

- 1) Review Ch. 3 Test
- 2) Objectives:
  - angles measuring angles in ~~degrees~~ radians
  - angles
- 3) Homework: pp. 261 #s 1-28, read vocabulary in: beginning of 4.1.

# HL Precalc

## Agenda:

- 1) ?'s on homework.
- 2) Objectives
  - Arc Length
  - Linear & Angular Speed
- 3) Homework: pp. 262-264 #s 44, 48, 54, 58, 64, 79, 88, 90, 94, 104, 106, 108, 110, 120.

## Algebra II (5 & 6)

11/8

### Agenda:

- 1) ?'s on homework
- 2) Objectives
  - Matrix addition & subtraction
  - Scalar multiplication
- 3) Homework: pp. 190-193 #s 1-4, 8, 10, 14, 18, 19, 23-30, 32-35

## 4. PreCalc

11/9

### Agenda:

- 1) ?'s on homework
- 2) Objectives:
  - Trigonometric functions
  - The unit circle
- 3) Homework: pp. 270-272 #s 12, 18-38 even, 42, 46, 50, 58, 68, 76, 78, 90

## Algebra II (per 5)

- 1) ?'s on homework
- 2) Objectives
  - matrix multiplication
- 3) Homework: pp 199-202 #s 1-5, 14, 16, 18, 20, 24, 28, 30, 32, 38, 40, 42, 45

## Algebra II (per. 6)

11/9

### Agenda:

#### 1) Objectives

- matrix addition & subtraction
- scalar multiplication

#### 2) Homework: pp. 190-193 #s 1-4, 8, 10, 14, 18, 19, 23-30, 32-35

## HL Precalc

11/10

### Agenda:

#### 1) ?'s on homework

#### 2) Objectives

- properties of trigonometric functions
- right triangle trigonometry
- trigonometric identities

#### 3) Homework: pp. 280-281 #s 8, 12, 13, 16, 18, 22, 26, 30, 32, 54, 56, 58, 62, 65, 68-72.

## Algebra II per 5

### Agenda:

#### 1) Warm-Up

#### 2) ?'s on homework

#### 3) Objectives:

- determinants
- Cramer's Rule

#### 4) Homework: pp. 207-209 #s 1-6, 11, 14, 18, 20, 21, 23-28, 30, 40, 45, 46

## H. PreCalc

11/11

### Agenda:

- 1) Warm-up
- 2) ?'s on homework
- 3) Objectives:
  - trig functions of any angle
  - Reference angles
- 4) Homework: pp. 282-283 #s 78, 79, 80, 83, 90  
pp. 289-291 #s 1a-2a even, 24, 28, 32, 34, 36,  
46, 50, 56, 62, 68, 72, 78, 82, 88, 104, 108.

## H. PreCalc

11/18

### Agenda:

- 1) ?'s on homework
- 2) Objectives:
  - evaluating inverse trig functions
  - graphing inverse trig functions.
- 3) Homework: pp. 322-324 #s 6, 12, 14, 16, 24,  
34, 40, 42, 46, 50, 58, 60, 70, 86, 90, 100.



## H. Precalc

11/18

### Agenda:

- 1) ?'s on homework
- 2) Objectives
  - Review graphs of sine & cosine
  - graph of tangent
- 3) Homework: Continued from yesterday. & MEMORIZE pg. 310.

## Alg II Per. 5

### Agenda:

- 1) Warm-Up
- 2) Return & go over quiz
- 3) Objectives
  - Review for Ch. 3 Test
- 4) Homework: STUDY & do pg. 222-226 #s 1-34 even

## Alg II Per. 6

### Agenda:

- 1) Warm-Up
- 2) ?'s on homework
- 3) Objectives
  - Review inverse matrices
  - Inverse matrices in calculator
- 4) Homework: pg. 219 #s 1-4 & pp. 222-226 #s 1-34 even

H. Precalc

11/21

Agenda:

- 1) Return quiz & go over
- 2) Objectives:
  - evaluating & graphing inverse trig functions
- 3) Homework: STUDY & pp. 322-324 #s 6, 12, 14, 16, 24, 34, 40, 42, 46, 50, 58, 60, 70, 86, 90, 100

Alg. II Period 5 & 6

11/22

Agenda:

- 1) Chapter 3 Test

H. Precalc

Agenda:

- 1) Chapter 4 Test

## 11/28

## Agenda

- Alg II (5 & 6)

Agenda:

- $$= \cancel{\cos \theta + \sin \theta} - \cancel{\cos^2 \theta - \sin^2 \theta}$$

59



## H. Precalc

11/29

### Agenda:

- 1) ?'s on homework
- 2) Objectives:
  - Verifying trig identities
  - Solving trig equations
- 3) Homework: pp. 362-364 #s 14, 32, 36, 42, 50, 56, 78, 84, 86  
pp. 373-375 #s 12, 20, 26, 34, 44, 49, 62, 68, 74, 80, 96, 98.

## Alg II (5&6)

### Agenda:

- 1) ?'s on homework
- 2) Objectives:
  - Graphing quadratic ~~eq~~ functions in ~~int~~ vertex & intercept forms
  - FOIL method
- 3) Homework: pp. 249-251 #s 6, 8, 9, 12, 14, 20, 24, 30, 34, 40, 50, 53, 56.

$$\begin{aligned} 0 &= a(x-33)^2 + 5 \\ 0 &= a(x^2 - 66x + 1089) + 5 \\ 0 &= a(x^2 - 66x + 1089) + 5 \\ 0 &= 1089a + 5 \quad a = \frac{-5}{1089} \end{aligned}$$

## H. Precalc.

### Agenda:

- 1) ?'s on homework
- 2) Objectives:
  - Solving trig equations cont'd.
  - Sum & difference formulas

3) Homework: pp. 381-383 #s 12, 18, 24, 32, 38, 42, 52, 55, 60, 68, 78, 82, 106.

## H. Algebra II

### Agenda:

- 1) ?'s on homework
- 2) Objectives:
  - Factoring quadratic equations
  - Finding zeros of quadratic equations

3) Homework: pp. 255-258 #s 4, 9, 10, 13, 16, 19, 24, 32, 37, 42, 48, 52, 57, 60, 64, 68, 70, 72

$$\frac{\frac{1}{\sin} + \frac{1}{\cos}}{\sin + \cos} = \frac{\frac{\cos + \sin}{\sin \cos}}{\sin + \cos} = \frac{\cos + \sin}{\sin \cos (\sin + \cos)} = \frac{1}{\sin \cos}$$

$$\frac{1}{\sin} \cdot \frac{1}{\cos} = \csc + \sec$$

$$\cot + \tan$$

$$\sin^2 - \cos^2 = \csc \sin - \cos \csc + \sec \sin + \sec$$

$$= -1 + \frac{\cos}{\sin} + \frac{\sin}{\cos} - 1$$

$$= \cot + \tan - 1$$

$$\sin^2 + \cos^2 = 1$$

$$\frac{1 + 2 \sin \cos}{1 + \cot + \tan + 1} = \frac{1 + 2 \sin \cos}{2 + \cot + \tan}$$

$$\frac{\sin^2 + 2 \sin \cos + \cos^2}{\sin \csc + \cos \csc + \sec \sin + \sec \cos}$$

$$\sin^2 + \cos^2 = 1$$

$$\sin^2 - \cos^2$$

12/2

## H. Precalc

### Agenda:

- 1) Quiz on chapter 5
- 2) ~~Multiple~~ ~~an~~ Objectives:
  - Multiple angle and product-to-sum formulas cont'd.
- 3) Homework: pp. 390-393 #s 22, 26, 30, 34, 44, 46, 52, 56, 62, 66, 72, 79, 86, 90, 100, 106, 112, 122, 130.

## H. Algebra II

### Agenda:

- 1) Quiz on 4.1-4.4
- 2) Objectives
  - Solving  $ax^2+bx+c$  cont'd.
- 3) Homework: pp. 263-265 #s 4, 10, 12, 14, 20, 24, 30, 32, 38, 42, 48, 52, 57, 60, 62, 64, 66,

12/5

## H. Precalc

### Agenda:

- 1) ?'s on homework
- 2) Objectives:
  - Law of Sines
  - Area of an Oblique Triangle
- 3) Homework: pp 410-412 #s 8, 12, 20, 28, 32, 34, 36, 43, 44, 49, 56

## H. Algebra II (5&6)

### Agenda:

- 1) ?'s on homework
- 2) Objectives:
  - Finding square roots
- 3) Homework

12/6

#### H. PreCalc

##### Agenda:

- 1) Return & Review Ch. 5 Quiz
- 2) ?'s on homework
- 3) Objectives:
  - Law of Cosines
  - Heron's Area Formula
- 4) Homework: pp. 417-419 #s 10, 14, 20, 26, 32, 36, 38, 46-48, 50, 56, 62, 64

#### H. Algebra II

##### Agenda:

- 1) ?'s on homework
- 2) Objectives:
  - The imaginary unit  $i$
  - Complex number operations
- 3) Homework: pp. 279-282 #s 4, 8, 10, 12, 16, 20, 24, 28, 30, 36, 38, 40, 44, 48, 61, 66, ~~74~~



12/7

### H. Precalc

#### Agenda:

1) Warm-up

2) ?'s on homework

3) Objectives:

• Vectors: operations & unit vectors

4) Homework: pp. 429-430 #s 14, 16, 22, 26,  
28, ~~30~~, 34, 38, 40, 44, 48, 50,

### H. Algebra II (5 & 6)

#### Agenda:

1) Return & Review 4.1-4.4 quiz

2) ?'s on homework

3) Objectives:

• Completing the Square

4) Homework: pp. 288-290 #s 8, 12, 14, 17,  
24, 30, 32, 36, ~~3~~ 42, 46, 56, ~~60~~ 61, 62,  
64, 67

12/8

#### H. Precalc

##### Agenda:

- 1) ?'s on homework
- 2) Objectives:
  - Standard Unit Vectors
  - Vector applications
- 3) Homework: pp. 430-433 #s 66, 70, 74, 76, 80, 84, 88, 97, 101, 112, 116, 124

#### H. Algebra II (5 & 6)

##### Agenda:

- 1) ?'s on homework
- 2) Objectives:
  - The quadratic formula
  - The discriminant
- 3) Homework: pp. 296-299 #s 4, 5, 12, 14, 20, 26, 32, 39, 42, 52-54, 56, 64, 67, 68, 70, 72

12/9

### H. Precalc

#### Agenda:

- 1) Quiz on 6.1-6.3
- 2) Warm-up exercises
- 3) ?'s on homework
- 4) Homework: continued from 12/8.

### H. Algebra II

#### Agenda:

- 1) ?'s on homework
- 2) objectives:
  - Graphing and solving quadratic inequalities.
- 3) Homework: pp. 304-307 #s 3-5, 8, 14, 22, 24, 28, 34, 36, 42, 44, 48, 56, 60, 65, 69, 70, 72, 76, 77



w/u: pg 430 # ~~49~~ 77

12/12

## II. Precalc

### Agenda:

- 1) Warm-up
- 2) Objectives:
  - Dot products
  - Vector Components
- 3) Homework: pp. 440-441 #s 10, 12, 16, 18, 21, 24, 26, 34, 36, 40, 44, 46, 48, 52, 58, 60, 66

## II. Algebra II (5 & 6)

### Agenda:

- 1) ?'s on homework
- 2) Objectives:
  - Writing quadratic functions & models
- 3) Homework: pp. 312-315 #s 4, 8, 14, 16, 17, 20, 24, 28, 34, 38, 40, 45, 46, 47, 50, 52

12/13

## H. PreCalc

### Agenda:

1) ?'s on homework

2) Objectives:

- Dot product applications
- Work

3) Homework: pp. 441-442 #s 70, 72, 73, 74,  
76, 78, 80, 84

12/15

## H. Precalc

Agenda: review quizzes

- 1) Donuts! ☺
- 2) ~~Trig~~ Objectives:
  - Trigonometric form of complex #s
- 3) NO homework!

## H. Alg. II (5 & 6)

Agenda:

- 1) Candy! ☺
- 2) Objectives: <sup>writing</sup>
  - Quadratic functions
- 3) Homework: pp. 312-315 #s 4, 10, 16, 18, 24, 30, 34, 38, 45, 46, 50, 52

# Appendix C

## Lecture Note Samples

### I. Algebra II

11/10

## Determinants

- Each square matrix has <sup>a</sup>determinant, denoted  $\det A$  or  $|A|$ .

•  $2 \times 2$ :

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

•  $3 \times 3$ :

- rewrite the first 2 columns to the right of the determinant.
- subtract the sum of the **Red products** from the sum of the **blue products**.

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} =$$

$$\begin{vmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{vmatrix} = (aei + bfg + cdh) - (gec + hfa + idb)$$



### Area of a Triangle

- The area of a triangle with vertices  $(x_1, y_1), (x_2, y_2), (x_3, y_3)$  is

$$\text{Area} = \pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

(Choose + or - to make the area positive.)

Cramer's Rule : use determinants to solve systems of linear equations.

- Linear system  $\Rightarrow$  Coefficient matrix

$$ax + by = e$$

$$cx + dy = f$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

- If  $\det A \neq 0$ , then the system has exactly one solution:

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\det A}$$

$$y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\det A}$$

#### 4.2 Quadratic Functions in Vertex/Intercept Form 11/29

- Vertex Form :  $y = a(x-h)^2 + k$ 
  - Vertex =  $(h, k)$
  - Axis of Symmetry :  $x = h$
  - Opens up if  $a > 0$  & down if  $a < 0$ .
- Intercept Form :  $y = a(x-p)(x-q)$ 
  - X-intercepts are  $p$  and  $q$
  - Axis of Symmetry :  $x = \frac{p+q}{2}$
  - Opens up if  $a > 0$  & down if  $a < 0$ .

- To multiply 2 expressions that each have 2 terms, use FOIL.

First

Outside

Inside

Last

$$(a+b)(a+c) = a^2 + ac + ab + bc$$



## Factoring

11/30

- To factor  $x^2+bx+c$ , find 2 numbers that multiply to give you  $c$  and add to give you  $b$ .  
(watch for signs, start by listing factors of  $c$ ).

Shortcuts:

- 1) Two squares:  $a^2-b^2=(a+b)(a-b)$
- 2) Perfect square:  $a^2+2ab+b^2=(a+b)^2$   
 $a^2-2ab+b^2=(a-b)^2$

Zeros: the solutions of quadratic equations are called zeros, or roots.

If  $(x-3)(x+2)=0$ , then either  
 $x-3=0$  or  $x+2=0$ .

12/6

### The Imaginary Unit $i$

- $i = \sqrt{-1}$

- $i^2 = -1$

① If  $r$  is a positive real number, then

$$\sqrt{-r} = i\sqrt{r}$$

②  $(i\sqrt{r})^2 = -r$

- Complex numbers in standard form :

$$a + bi$$

### Sums & Differences

- Sum of Complex Numbers :

$$(a+bi) + (c+di) = (a+c) + (b+d)i$$

- Difference of Complex Numbers :

$$(a+bi) - (c+di) = (a-c) + (b-d)i$$

### Complex Conjugates

$$a+bi, a-bi$$

- To write the quotient of 2 complex numbers in standard form, multiply the numerator and denominator by the complex conjugate of the denominator.

### Complex Plane

- Horizontal axis : Real part
- Vertical axis : Imaginary part

### Absolute Value

The absolute value of a complex number

$$z = a+bi \text{ is}$$

$$|z| = \sqrt{a^2 + b^2}$$

12/7

### Completing the Square

Turns trinomials into perfect squares  $(x+b)^2$ .

To complete the square for  $x^2+bx$ ,  
add  $(\frac{b}{2})^2$ .

$$x^2+bx+(\frac{b}{2})^2=(x+\frac{b}{2})(x+\frac{b}{2})=(x+\frac{b}{2})^2$$

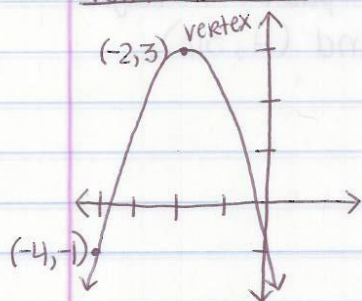
To write a quadratic function in vertex form,  
use completing the square.



12/15

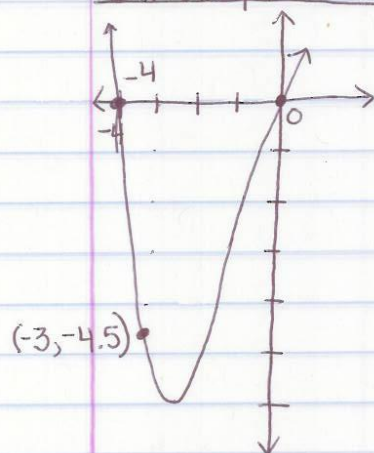
## Writing Quadratic Functions & Models

### Vertex Form



Write a quadratic function for the parabola.

### Intercept Form



Write a quadratic function for the parabola.

Given:

Use:

How?

Vertex &  
1 point

Vertex  
form

① Plug in vertex  
② Use point to solve for  $a$ .

X-intercept(s)  
& 1 point

Intercept  
form

① Plug in x-intercepts  
② Use point to solve for  $a$ .

3 points

Standard  
form

① Plug in points  
② Solve system of equations.

# Appendix C

## Lecture Note Samples

### II. Precalculus

### 3.4 Solving Exponential & Logarithmic Equations

10/24

#### • Properties (for $a > 0$ & $a \neq 1$ ):

- $a^x = a^y$  IFF  $x = y$
- $\log_a x = \log_a y$  IFF  $x = y$
- $\log_a a^x = x$
- $a^{\log_a x} = x$

} one-to-one properties

} inverse properties

#### • Strategies for solving equations:

- Rewrite an exponential equation in logarithmic form & apply inverse property of logarithmic functions
- Rewrite a logarithmic equation in exponential form & apply inverse property of exponential functions
- Rewrite the original equation in a form that lets you use the one-to-one properties



## 4.2 Trigonometric Functions

11/9

• Let  $t$  be a real number and let  $(x, y)$  be the point on the unit circle corresponding to  $t$ . Then

1)  $\sin(t) = y$

2)  $\cos(t) = x$

3)  $\tan(t) = \frac{y}{x}, x \neq 0$

4)  $\csc(t) = \frac{1}{y}, y \neq 0$

5)  $\sec(t) = \frac{1}{x}, x \neq 0$

6)  $\cot(t) = \frac{x}{y}, y \neq 0$

### Properties of Sine & Cosine

• Domain:  $\mathbb{R}$

• Range:  $-1 \leq \sin(t), \cos(t) \leq 1$

• periodic

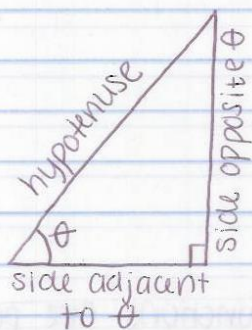
- A function  $f$  is periodic when there exists a positive real number  $c$  such that  $f(t+c) = f(t)$
- The least number  $c$  for which  $f$  is periodic is called the period of  $f$ .

### Even & Odd Properties

- Cosine & secant are even
  - $\cos(-t) = \cos(t)$       •  $\sec(-t) = \sec(t)$
- Sine, cosecant, tangent, & cotangent are odd.
  - $\sin(-t) = -\sin(t)$       •  $\csc(-t) = -\csc(t)$
  - $\tan(-t) = -\tan(t)$       •  $\cot(-t) = -\cot(t)$

## Right Angle Trigonometry

11/10



Note that since  $0^\circ < \theta < 90^\circ$ , all the trig functions are positive.

- Let  $H$  = length of the hypotenuse,  
 $O$  = length of the side opposite  $\theta$ ,  
 $A$  = length of the side adjacent to  $\theta$ .

$$\sin \theta = \frac{O}{H}$$

$$\csc \theta = \frac{H}{O}$$

$$\cos \theta = \frac{A}{H}$$

$$\sec \theta = \frac{H}{A}$$

$$\tan \theta = \frac{O}{A}$$

$$\cot \theta = \frac{A}{O}$$



## Trigonometric Identities

11/10

### • Reciprocals

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

### • Quotients

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

### • Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1 \quad (\sin^2 \theta = (\sin \theta)^2)$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

#### 4.4 Trigonometric Functions of Any Angle

11/14

- Let  $\theta$  be an angle in standard position with  $(x, y)$  a point on the terminal side of  $\theta$  and  $r = \sqrt{x^2 + y^2} \neq 0$ . Then

$$\sin \theta = \frac{y}{r}$$

$$\csc \theta = \frac{r}{y}, y \neq 0$$

$$\cos \theta = \frac{x}{r}$$

$$\sec \theta = \frac{r}{x}, x \neq 0$$

$$\tan \theta = \frac{y}{x}, x \neq 0$$

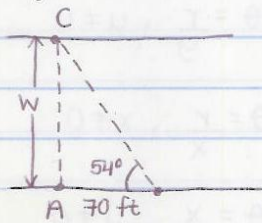
$$\cot \theta = \frac{x}{y}, y \neq 0$$

Note:

quadrant II:	quadrant I:
$\sin \theta: +$	$\sin \theta: +$
$\cos \theta: -$	$\cos \theta: +$
$\tan \theta: -$	$\tan \theta: +$
quadrant III:	quadrant IV:
$\sin \theta: -$	$\sin \theta: -$
$\cos \theta: -$	$\cos \theta: +$
$\tan \theta: +$	$\tan \theta: -$

### Warm-up

- A biologist wants to know the width  $w$  of a river. From point A, the biologist walks downstream 70 feet and sights to point C from an angle of  $54^\circ$ . How wide is the river?



## 5.1 Fundamental Identities

11/28

### • Cofunction Identities

$$\cdot \sin\left(\frac{\pi}{2} - u\right) = \cos u$$

$$\cdot \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\cdot \tan\left(\frac{\pi}{2} - u\right) = \cot u$$

$$\cdot \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\cdot \sec\left(\frac{\pi}{2} - u\right) = \csc u$$

$$\cdot \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

## 5.2 Verifying Trig Identities

11/29

Hints:

- 1) Factor, add fractions, Square binomials, create a common denominator.
- 2) Use the fundamental identities
- 3) When all else fails, try converting all terms to sine and cosine.



12/1

### Double Angle Formulas

$$\bullet \sin 2x = 2 \sin x \cos x$$

$$\bullet \cos 2x = \cos^2 x - \sin^2 x$$

$$= 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

$$\bullet \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

### Power Reducing Formulas

$$\bullet \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\bullet \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\bullet \tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$



### Half Angle Formulas

$$\bullet \sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\bullet \cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\bullet \tan \frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

### Product to Sum Formulas

$$\bullet \sin u \sin v = \frac{1}{2} [\cos(u-v) - \cos(u+v)]$$

$$\bullet \cos u \cos v = \frac{1}{2} [\cos(u-v) + \cos(u+v)]$$

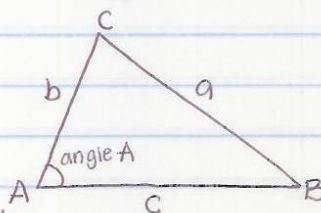
$$\bullet \sin u \cos v = \frac{1}{2} [\sin(u+v) + \sin(u-v)]$$

$$\bullet \cos u \sin v = \frac{1}{2} [\sin(u+v) - \sin(u-v)]$$

Law of Sines

If ABC is a triangle with sides  $a$ ,  $b$ , and  $c$ , then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Need: ① 2 angles and any side

② 2 sides and an angle opposite one of them.

The Ambiguous Case

Consider a triangle where you are given  $a$ ,  $b$ , and  $A$ . (Note  $h = b \sin A$ )

	acute	acute	acute	acute	obtuse	obtuse
Necessary Cond.	$a < h$	$a = h$	$a \geq b$	$h < a < b$	$a \leq b$	$a > b$
Possible Triangles	0	1	1	2	0	1

12/6

Law of Cosines

Standard Form

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Alternative Form

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Note: It is usually helpful to solve for the largest angle first, then use the simpler Law of Sines to find the other angles.

Heron's Area Formula

For any triangle with sides of lengths  $a$ ,  $b$ , and  $c$ ,

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+c}{2}$$



12/7

Component Form of a Vector

The component form of a vector with initial point  $P(p_1, p_2)$  and terminal point  $Q(q_1, q_2)$  is

$$\vec{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle = \mathbf{v}$$

The magnitude (length) of  $\mathbf{v}$  is

$$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$$

If  $\|\mathbf{v}\| = 1$ , then  $\mathbf{v}$  is a unit vector.

$\|\mathbf{v}\| = 0$  IFF  $\mathbf{v}$  is the zero vector  $\mathbf{0}$ .

- Two vectors  $\mathbf{u} = \langle u_1, u_2 \rangle$  and  $\mathbf{v} = \langle v_1, v_2 \rangle$  are equal IFF  $u_1 = v_1$  and  $u_2 = v_2$

### Vector Addition & Scalar Multiplication

Let  $u = \langle u_1, u_2 \rangle$  and  $v = \langle v_1, v_2 \rangle$  and let  $k$  be a scalar.

$$u + v = \langle u_1 + v_1, u_2 + v_2 \rangle$$

$$ku = k\langle u_1, u_2 \rangle = \langle ku_1, ku_2 \rangle$$

### Properties

Let  $u, v$ , and  $w$  be vectors and  $c$  and  $d$  be scalars.

①  $u + v = v + u$

②  $(u + v) + w = u + (v + w)$

③  $u + 0 = u$

④  $u + (-u) = 0$

⑤  $c(du) = (cd)u$

⑥  $(c + d)u = cu + du$

⑦  $c(u + v) = cu + cv$

⑧  $1(u) = u, 0(u) = 0$

⑨  $\|cv\| = |c| \|v\|$

### Unit Vectors

Unit vectors are vectors that have magnitude 1.

To find a unit vector, divide the vector by its magnitude:

$$\frac{v}{\|v\|} = \left( \frac{1}{\|v\|} \right) v = \text{unit vector in the direction of } v.$$

12/8

Standard Unit Vectors

$$\vec{i} = \langle 1, 0 \rangle \quad \vec{j} = \langle 0, 1 \rangle$$

Any vector can be written as a linear combination of  $\vec{i}$  and  $\vec{j}$ :

$$\begin{aligned} \vec{v} &= \langle v_1, v_2 \rangle \\ &= v_1 \langle 1, 0 \rangle + v_2 \langle 0, 1 \rangle \\ &= v_1 \vec{i} + v_2 \vec{j} \end{aligned}$$

Direction Angles

If  $\vec{u}$  is a unit vector such that  $\theta$  is the angle from the positive x-axis to  $\vec{u}$ , then

$$\vec{u} = (\cos \theta) \vec{i} + (\sin \theta) \vec{j},$$

and  $\theta$  is the direction angle of  $\vec{u}$ .

Suppose  $\vec{v}$  is not a unit vector. Then

$$\vec{v} = \|\vec{v}\|(\cos \theta) \vec{i} + \|\vec{v}\|(\sin \theta) \vec{j}$$

The direction angle  $\theta$  for  $\vec{v}$  is

$$\tan \theta = \frac{\|\vec{v}\| \sin \theta}{\|\vec{v}\| \cos \theta} = \frac{b}{a}$$



12/12

Dot Products

The dot product of  $\vec{u} = \langle u_1, u_2 \rangle$  and  $\vec{v} = \langle v_1, v_2 \rangle$  is:  $\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2$

Properties

Let  $\vec{u}, \vec{v}$ , and  $\vec{w}$  be vectors and  $c$  be a scalar.

$$\textcircled{1} \vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$$

$$\textcircled{2} \vec{0} \cdot \vec{v} = 0$$

$$\textcircled{3} \vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$$

$$\textcircled{4} \vec{v} \cdot \vec{v} = \|\vec{v}\|^2$$

$$\textcircled{5} c(\vec{u} \cdot \vec{v}) = c\vec{u} \cdot \vec{v} = \vec{u} \cdot c\vec{v}$$

Angle Between 2 Vectors

If  $\theta$  is the angle between 2 vectors  $\vec{u}$  and  $\vec{v}$ , then

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$$

### Orthogonal Vectors

The vectors  $\vec{u}$  and  $\vec{v}$  are orthogonal when  $\vec{u} \cdot \vec{v} = 0$ .

### Vector Components

Let  $\vec{u}$  and  $\vec{v}$  be vectors such that

$$\vec{u} = \vec{w}_1 + \vec{w}_2$$

where  $\vec{w}_1$  and  $\vec{w}_2$  are orthogonal &  $\vec{w}_1$  is parallel to  $\vec{v}$ . Then  $\vec{w}_1$  &  $\vec{w}_2$  are called vector components of  $\vec{u}$ .

$\vec{w}_1$  is the projection of  $\vec{u}$  onto  $\vec{v}$ :

$$\vec{w}_1 = \text{proj}_{\vec{v}} \vec{u}$$

Then

$$\vec{w}_2 = \vec{u} - \vec{w}_1.$$

### Projection of $\vec{u}$ onto $\vec{v}$

The projection of  $\vec{u}$  onto  $\vec{v}$  is given by

$$\text{proj}_{\vec{v}} \vec{u} = \left( \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2} \right) \vec{v}$$



# Appendix D

## Handouts/Review Sheets

### I. Precalculus

## Practice Problems

10/24

① Solve:  $\log_{10} x = -2$

② Solve:  $4(3^x) = 64$

③ Solve:  $e^{2x} - 7e^x + 12 = 0$   
(Hint: factor!)

④ Solve:  $\log_{10}(x+4) + \log_{10}(x+1) = 1$

# Ch. 2 Review

- ① Write the equation of the quadratic function (in standard form) whose vertex is  $(3, -6)$  and who passes through the point  $(0, 3)$ .

$$y = a(x-3)^2 - 6$$

$$3 = a(0-3)^2 - 6$$

$$3 = 9a - 6$$

$$9 = 9a$$

$$a = 1$$

$$y = (x-3)^2 - 6$$

- ② Use long division to divide  $3x^3 + 4x - 1$  by  $x^2 + 1$ .

$$\begin{array}{r} 3x + \frac{x-1}{x^2+1} \\ x^2+1 \overline{) 3x^3+0x^2+4x-1} \\ \underline{-3x^2 \phantom{+0x} - 3x} \phantom{-1} \\ x-1 \end{array}$$

$$3x + \frac{x-1}{x^2+1}$$

- ③ Find the vertex and x-intercepts (zeros) of  $f(x) = x^2 + 4x + 3$ . (hint: to find vertex, complete the square and put in standard form!)

$$x\text{-int: } f(x) = (x+3)(x+1)$$

$$x = -3 \quad x = -1$$

$$(-1, 0) \text{ and } (-3, 0)$$

$$(x^2 + 4x + 4) + 3 - 4$$

$$= (x+2)^2 - 1$$

$$\text{Vertex} = (-2, -1)$$

- ④ Use synthetic division to evaluate  $f(-2)$  for  $f(x) = 3x^4 - 6x^2 + 5x - 1$  (hint: Remainder theorem!)

$$\begin{array}{r} -2 \overline{) 3 \ 0 \ -6 \ 5 \ -1} \\ \underline{\downarrow -6 \ 12 \ -12 \ 14} \\ 3 \ -6 \ 6 \ -7 \ 13 \end{array}$$

$$13$$

# Review

- ⑤ Write the quotient in standard form ( $a+bi$ ):

$$\frac{2i-1}{3i+2}$$

$$\frac{(2i-1)(3i-2)}{(3i+2)(3i-2)} = \frac{6i^2 - 7i + 2}{9i^2 - 4}$$

$$= \frac{-4 - 7i}{-13}$$

$$= \boxed{\frac{4}{13} + \frac{7}{13}i}$$

- ⑥ Solve  $x^2 - 2x + 8 = 0$ .  
(hint: quadratic formula!)

$$x = \frac{2 \pm \sqrt{4 - 4(8)}}{2}$$

$$= \frac{2 \pm \sqrt{-28}}{2}$$

$$= \frac{2 \pm 2\sqrt{-7}}{2}$$

$$= \frac{2 \pm 2\sqrt{7}i}{2}$$

$$= \boxed{1 \pm \sqrt{7}i}$$

- ⑦ Sketch the graph of the Rational function  $f(x) = \frac{x^2+2}{x-1}$ .  
(hint: don't forget about slant asymptotes!)

\* Go through the 7 steps!

VA:  $x=1$

HA: none

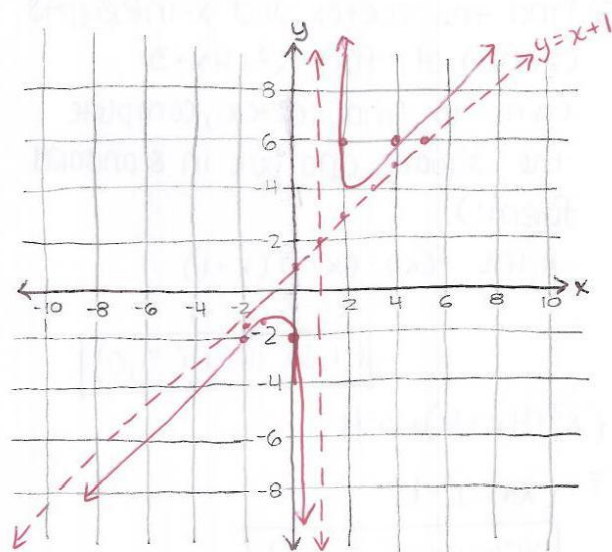
SA:  $x-1 \overline{) x^2 + 0x + 2} : y=x+1$

$$\begin{array}{r} x+1 + \frac{3}{x-1} \\ x-1 \overline{) x^2 + 0x + 2} \\ \underline{-x^2 - x} \phantom{+ 2} \\ x+2 \\ \underline{-x-1} \\ 3 \end{array}$$

x-int: none

y-int:  $(0, -2)$

$(2, 6)$   $(-2, -2)$   $(-1, -\frac{3}{2})$   $(4, 6)$



# Ch. 3 Review

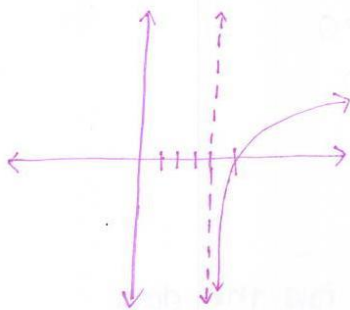
- ① Find the domain, vertical asymptote, & x-intercept.  
Sketch the graph of the function.

$$f(x) = \ln(x-4)$$

$$\text{domain: } (4, \infty)$$

$$\text{VA: } x=4$$

$$\text{x-int: } (5, 0)$$



- ② Change to base 10.

$$\log_{12} 64$$

$$\frac{\log_{10} 64}{\log_{10} 12}$$

$$\log_{10} 12$$

- ③ Expand as a combination of multiple logarithms.

$$\ln\left(\frac{5\sqrt{x}}{6}\right)$$

$$\ln 5 + \ln \sqrt{x} - \ln 6$$

$$\ln 5 + \frac{1}{2} \ln x - \ln 6$$

- ④ Condense to a single logarithm.

$$\ln x + \ln(2x-3) - \ln(x+2)$$

$$\ln\left(\frac{x(2x-3)}{x+2}\right)$$



⑤ Solve for  $x$ .

a)  $\log_{10}(x-4) = 2$

$$10^2 = x-4$$

$$100 = x-4$$

$$x = 104$$

c)  $\log_{10} x + \log_{10}(x-9) = 1$

$$\log_{10}(x(x-9)) = 1$$

$$10 = x^2 - 9x$$

$$x^2 - 9x - 10 = 0$$

$$(x-10)(x+1) = 0$$

$$x = 10 \quad x \neq -1$$

b)  $\ln(x-2) + \ln(2x-3) = 2 \ln x$

$$\ln((x-2)(2x-3)) = \ln x^2$$

$$\ln(2x^2 - 7x + 6) = \ln x^2$$

$$2x^2 - 7x + 6 = x^2$$

$$x^2 - 7x + 6 = 0$$

$$(x-6)(x-1) = 0$$

$$x = 6$$

$$x \neq 1$$

d)  $2x \ln x - x = 0$

$$x(2 \ln x - 1) = 0$$

$$x(\ln x^2 - 1) = 0$$

$$\ln x^2 - 1 = 0$$

$$\ln x^2 = 1$$

$$e = x^2$$

$$x = \sqrt{e}$$

⑥ Find quadratic, exponential, & power models for the data.  
Use the  $r$ -values to determine which is the best fit.

$x$	$y$
0.9	0.016
1.0	0.019
1.5	0.040
1.8	0.061
2.1	0.087
2.2	0.092
2.4	0.111

Quadratic:  $y = 0.023x^2 - 0.013x + 0.008$

$$R^2 = .9985695$$

Exponential:  $y = 0.005(3.73)^x$

$$R^2 = .9917372$$

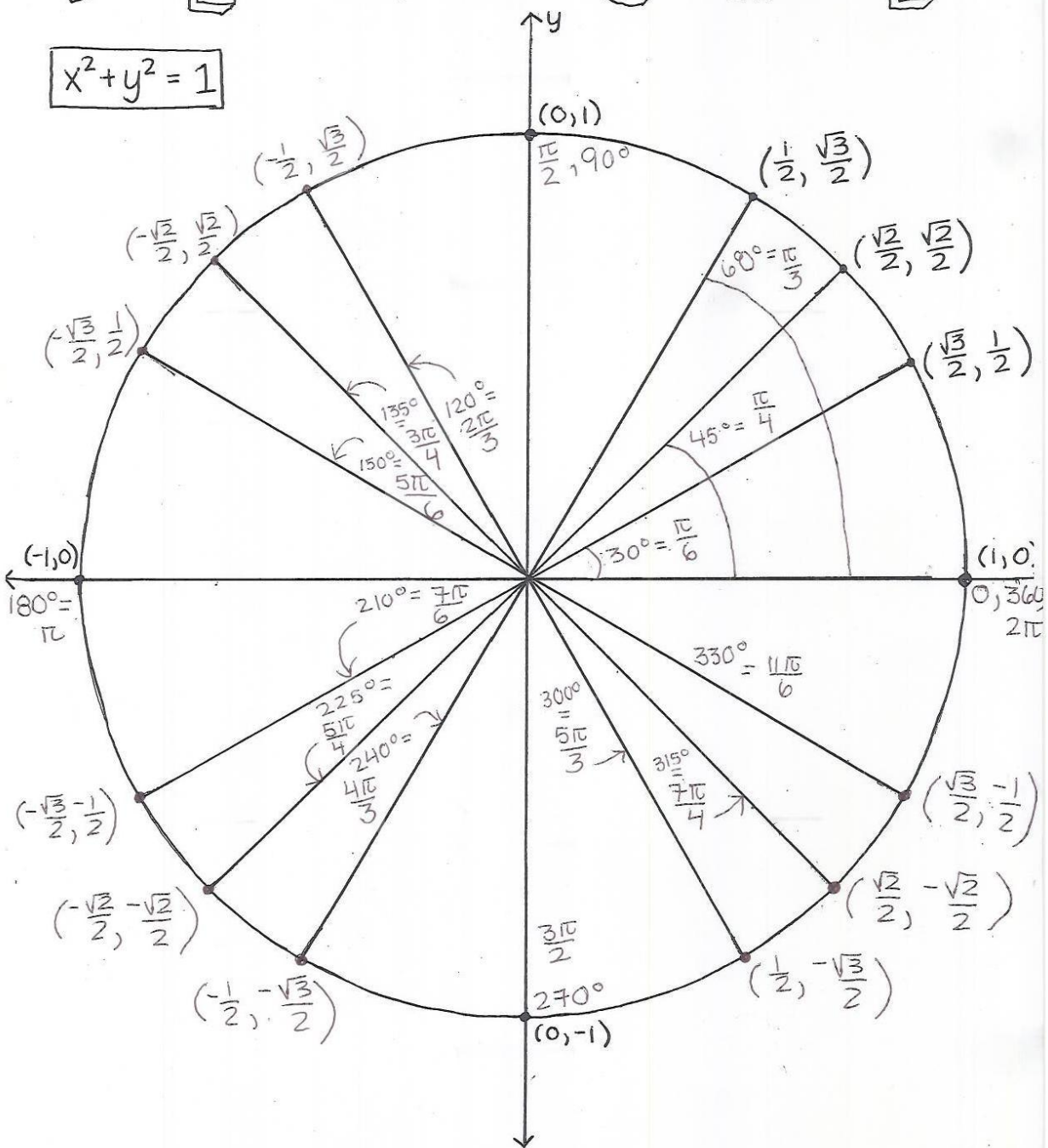
Power:  $y = 0.019x^2$

$$R^2 = .9989559$$

Power,  $y = 0.019x^2$ , is the best fit.

# THE UNIT CIRCLE

$$x^2 + y^2 = 1$$



# Appendix E

## Assessments

### I. Algebra II



Name: KEY

Period 5

# Quiz 10 possible points

For 1 and 2, use the given matrices to evaluate the expression.

$$A = \begin{bmatrix} 1 & -4 \\ 5 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -6 & -1 \\ 2 & 4 \end{bmatrix}$$

①  $3A + B$  (2 pts)

$$= \begin{bmatrix} 3 & -12 \\ 15 & 6 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -15 \\ 15 & 7 \end{bmatrix}$$

②  $C(B-A)$  (2 pts)

$$B-A = \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -4 \\ 5 & 2 \end{bmatrix}$$

$$= C \left( \begin{bmatrix} 1 & 1 \\ -5 & -1 \end{bmatrix} \right) = \begin{bmatrix} 1 & 1 \\ -5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -5 \\ -18 & -2 \end{bmatrix}$$

③ Solve using Cramer's Rule. (3 pts)

$$\begin{aligned} 6x - 2y &= -16 \\ -3x + 5y &= 16 \end{aligned} \quad A = \begin{bmatrix} 6 & -2 \\ -3 & 5 \end{bmatrix}$$

$$\det A = 30 - 6 = 24$$

$$x = \frac{\begin{vmatrix} -16 & -2 \\ 16 & 5 \end{vmatrix}}{24} = \frac{-80 + 32}{24} = \frac{-48}{24} = -2$$

$$y = \frac{\begin{vmatrix} 6 & -16 \\ -3 & 16 \end{vmatrix}}{24} = \frac{96 - 48}{24} = \frac{48}{24} = 2$$

$$(-2, 2)$$

④ The vertices of a sail for a sailboat are given by (0,2), (12,2) and (12,26), measured in feet. Find the area of the sail. (3 pts)

$$\pm \frac{1}{2} \begin{vmatrix} 0 & 2 & 1 \\ 12 & 2 & 1 \\ 12 & 26 & 1 \end{vmatrix} = \frac{312 - 24}{336}$$

$$= \pm \frac{1}{2} [(0 + 24 + 312) - (24 + 0 + 24)]$$

$$= \pm \frac{1}{2} (336 - 48) = \pm \frac{1}{2} (288) = 144 \text{ ft}^2$$

Name:

Period 6

KEY

## Quiz

For 1 and 2, use the given matrices to evaluate the expression.

$$A = \begin{bmatrix} 1 & -4 \\ 5 & 2 \end{bmatrix}$$

(2 pts)

$$B = \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -6 & -1 \\ 2 & 4 \end{bmatrix}$$

(2 pts)

①  $3A + B$

$$= \begin{bmatrix} 3 & -12 \\ 15 & 6 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -15 \\ 15 & 7 \end{bmatrix}$$

②  $C(B-A)$

$$B-A = \begin{bmatrix} 1 & 1 \\ -5 & -1 \end{bmatrix}$$

$$C(B-A) = \begin{bmatrix} -6 & -1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -5 & -1 \end{bmatrix} =$$

$$\begin{bmatrix} -1 & -5 \\ -18 & -2 \end{bmatrix}$$

(3 pts)

③ Find the determinant of the matrix.

$$\begin{vmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & 3 & -1 \end{vmatrix}$$

$$= (-1 + 0 + 18) - (-4 + 12 + 0)$$

$$= 17 - 8 = \boxed{9}$$

(3 pts)

④ The vertices of a sail for a sailboat are given by  $(0, 2)$ ,  $(12, 2)$ ,  $(12, 26)$ , measured in feet. Find the area of the sail.

$$A = \pm \frac{1}{2} \begin{vmatrix} 0 & 2 & 12 \\ 12 & 2 & 12 \\ 12 & 26 & 12 \end{vmatrix}$$

$$= \frac{1}{2} (0 + 24 + 312) - (24 + 0 + 24)$$

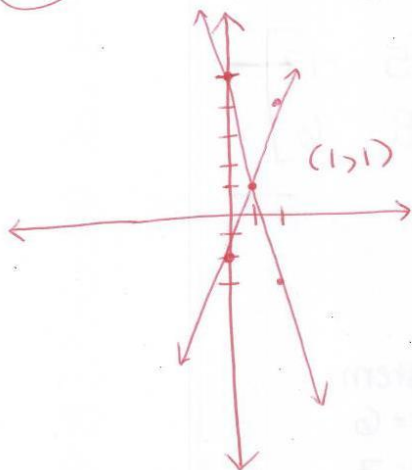
$$= \frac{1}{2} (336 - 48) = \frac{1}{2} (288) = \boxed{144}$$

Name: KEY

## Ch. 3 Test

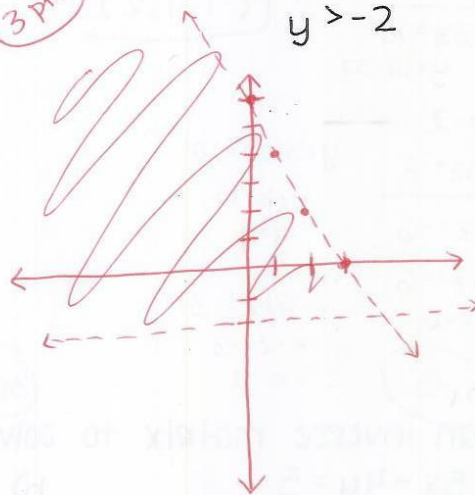
① Graph the linear system and estimate the solution.

(3pts)  $4x + y = 5$   $y = -4x + 5$   
 $3x - y = 2$   $y = 3x - 2$



② Graph the system of linear inequalities.

(3pts)  $2x + y < 6$   $y < -2x + 6$   
 $y > -2$



③ Evaluate the determinant.

(2pts)  $\begin{vmatrix} 4 & -1 & 2 \\ -3 & -2 & -1 \\ 0 & 5 & 1 \end{vmatrix}$

$$= (-8 + 0 - 30) - (0 - 20 + 3)$$

$$= -38 + 17 = \boxed{-21}$$

④ Find the area of the triangle with vertices  $(-4, 6)$ ,  $(0, 3)$ , and  $(6, 6)$ .

(2pts)  $\pm \frac{1}{2} \begin{vmatrix} -4 & 6 & 1 \\ 0 & 3 & 1 \\ 6 & 6 & 1 \end{vmatrix}$

$$= \pm \frac{1}{2} [(-12 + 36 + 0) - (18 - 24 + 0)]$$

$$= \pm \frac{1}{2} [24 + 6] = \frac{1}{2} (30) = \boxed{15 \text{ units}^2}$$

⑤ Solve the system. (2 pts)

$$x - y + z = -3$$

$$2x - y + 5z = 4$$

$$4x + 2y - z = 2$$

$$\begin{array}{r} 2x - y + 5z = 4 \\ + -2x + 2y - 2z = 6 \\ \hline y + 3z = 10 \\ y = 10 - 3z \end{array}$$

$$(-1, 4, 2)$$

$$\begin{array}{r} 4x + 2y - z = 2 \\ + -4x + 2y - 10z = -8 \\ \hline 4y - 11z = -6 \\ 4(10 - 3z) - 11z = -6 \\ 40 - 12z - 11z = -6 \\ -23z = -46 \\ z = 2 \end{array}$$

$$\begin{array}{l} y + 3(2) = 10 \\ y + 6 = 10 \\ y = 4 \end{array}$$

$$\begin{array}{l} x - 4 + 2 = -3 \\ x - 2 = -3 \\ x = -1 \end{array}$$

⑦ Use an inverse matrix to solve the system. (3 pts)

a)  $-5x - 4y = 5$

$4x + 3y = -3$

$$A = \begin{bmatrix} -5 & -4 \\ 4 & 3 \end{bmatrix} \quad |A| = -15 + 16 = 1$$

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} 3 & 4 \\ -4 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -5 \end{bmatrix}$$

⑥ Find  $C(A+B)$ . (2 pts)

$$A = \begin{bmatrix} 1 & -2 \\ 4 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 5 \\ -1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 3 & -3 \end{bmatrix} =$$

$$\begin{bmatrix} 5 & -12 \\ 8 & 6 \end{bmatrix}$$

(3 pts)

b)  $3x + 4y = 6$

$4x + 5y = 7$

$$A = \begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix} \quad |A| = 15 - 16 = -1$$

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} 5 & -4 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 6 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$



Name: (pers. 5 & 6)

12/2/2011

KEY

# Quiz

$$3(-3) = -9(2) = -18$$
$$2(3)(-3) = -18$$

① Graph the function. Label the vertex and axis of symmetry.

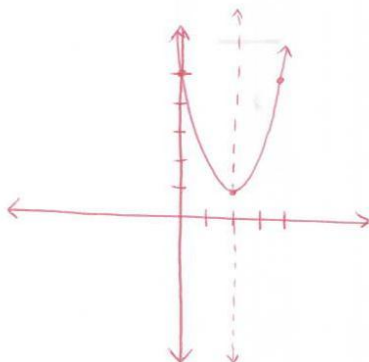
a)  $y = (x-2)^2 + 1$

vertex: (2, 1)

A.O.S:  $x = 2$

(0, 5)

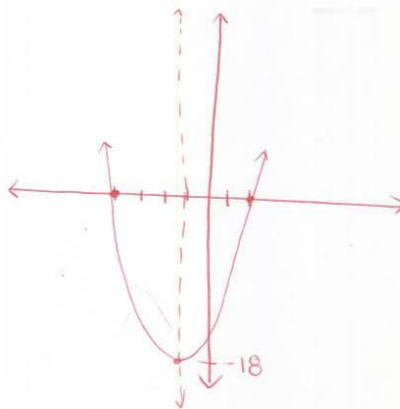
3pts



b)  $y = 2(x+4)(x-2)$

vertex: (-1, -18) A.O.S:  $x = -1$

3pts



② Solve for x.

$$x^2 - 3x - 40 = 0$$

$$(x-8)(x+5) = 0$$

$$x = 8 \quad x = -5$$

2pts

③ Factor  $3x^2 - 10x + 8$ .

$$3x^2 - 10x + 8 =$$

$$(3x+2)(x-4)$$

2pts

$$(3x-4)(x-2)$$

$$= 3x^2 - 6x - 4x + 8$$

$$= 3x^2 - 10x + 8$$

# Appendix E

## Assessments

### II. Precalculus

Name:

# Quiz

• Pick 4 of the following 5 problems. Show all work!

- ① Write the standard form of the quadratic function that has the given vertex and passes through the given point.

Vertex:  $(1, -2)$

Point:  $(-1, 14)$

- ② Find a polynomial function with the given zeros, multiplicities, and degree.

zero:  $-2$  multiplicity:  $2$

zero:  $-1$  multiplicity:  $1$

degree:  $3$

- ③ Sketch the graph of  $f(x) = 2x^3 - 6x^2$  by using the leading coefficient test, finding the zeros of the function, and plotting a few extra points.



④ Use long division to divide  $x^4 + 5x^3 + 6x^2 - x - 2$  by  $x + 2$ .

⑤ Use synthetic division to divide  $3x^3 - 17x^2 + 15x - 25$  by  $x - 5$ .

KEY

# Ch. 2 Test

24 pts  
total

Name: \_\_\_\_\_

- ① Write the equation of the quadratic function (in standard form) whose vertex is (2,3) and who passes through (0,2).

$$f(x) = a(x-2)^2 + 3$$

$$2 = a(0-2)^2 + 3$$

$$2 = 4a + 3$$

$$-1 = 4a$$

$$a = -1/4$$

$$f(x) = -\frac{1}{4}(x-2)^2 + 3$$

2 pts

- ② Identify the vertex and x-intercepts of

$$f(x) = x^2 - 4x - 5.$$

$$\text{vertex: } (x^2 - 4x + 4) - 5 - 4$$

$$= (x-2)^2 - 9$$

$$= (2, -9)$$

$$\text{x-int: } (x-5)(x+1)$$

$$x = 5 \quad x = -1$$

$$(5, 0) \quad (-1, 0)$$

2 pts

- ③ Find all the real zeros of  $f(x) = x^4 - x^3 - 2x^2$  and state their multiplicities.

$$0 = x^2(x^2 - x - 2)$$

$$0 = x^2(x-2)(x+1)$$

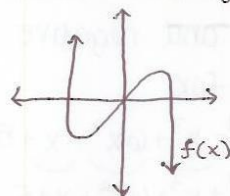
$$x = 0, \text{ mult. } 2$$

$$x = 2, \text{ mult. } 1$$

$$x = -1, \text{ mult. } 1$$

3 pts

- ④ Which polynomial could represent the graph?



1 pt

- a. 2<sup>nd</sup> degree, positive leading coeff.
- b. 2<sup>nd</sup> degree, negative leading coeff.
- c. 3<sup>rd</sup> degree, positive leading coeff.
- d. 3<sup>rd</sup> degree, negative leading coeff.

- ⑤ Use long division to divide  $4x^3 - 7x^2 - 11x + 5$  by  $4x + 5$ .

$$\begin{array}{r}
 x^2 - 3x + 1 \\
 4x+5 \overline{) 4x^3 - 7x^2 - 11x + 5} \\
 \underline{- 4x^3 + 5x^2} \phantom{+ 5x} \phantom{+ 5} \\
 -12x^2 - 11x \phantom{+ 5} \\
 \underline{- -12x^2 - 15x} \phantom{+ 5} \\
 4x + 5
 \end{array}$$

$$x^2 - 3x + 1$$

2pts

- ⑥ Use synthetic division to evaluate  $f(2)$  for  $f(x) = 2x^3 - 7x + 3$ .

$$\begin{array}{r|rrrr}
 2 & 2 & 0 & -7 & 3 \\
 & \downarrow & 4 & 8 & 2 \\
 \hline
 & 2 & 4 & 1 & 5
 \end{array}$$



$$f(2) = 5$$

2pts

- ⑦ Use Descartes Rule of Signs to determine the number of positive and negative real zeros for

$$f(x) = 2x^4 - x^3 + 6x^2 - x + 5$$

$$f(-x) = 2x^4 + x^3 + 6x^2 + x + 5$$

4, 2, or 0 Real positive zeros  
No Real negative zeros

2pts

- ⑧ Write the quotient in standard form:

$$\frac{2+3i}{4-2i}$$

$$\frac{(2+3i)(4+2i)}{(4-2i)(4+2i)} = \frac{8+16i+6i^2}{16-4i^2}$$

$$= \frac{2+16i}{20}$$

$$= \frac{1}{10} + \frac{4}{5}i$$

2pts

9) Solve the quadratic equation  $x^2+6x+10=0$ .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{36 - 40}}{2}$$

$$= \frac{-6 \pm \sqrt{-4}}{2}$$

$$= \frac{-6 \pm 2i}{2}$$

$$= \boxed{-3 \pm i}$$

2 pts

10) Sketch the rational function. Check for all asymptotes!

$$f(x) = \frac{2x^2 + 1}{x}$$

VA:  $x=0$

HA: none

SA:  $y=2x$

x-int: none

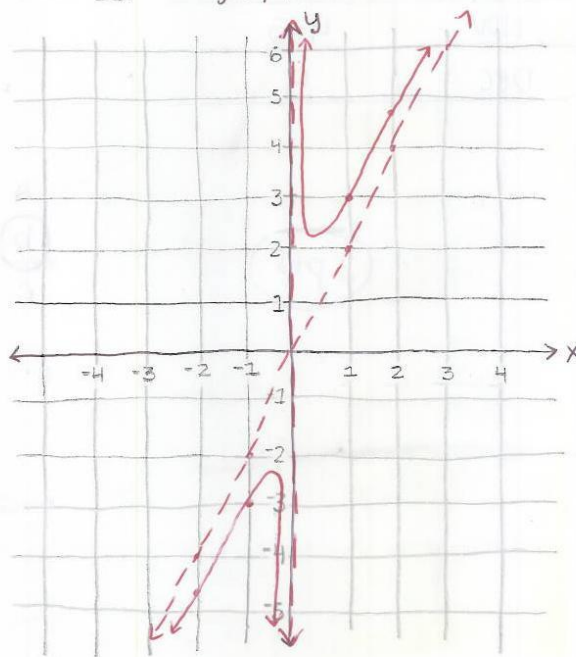
y-int: none

$(1, 3)$   $(2, \frac{9}{2})$

$(-1, -3)$   $(-2, -\frac{9}{2})$

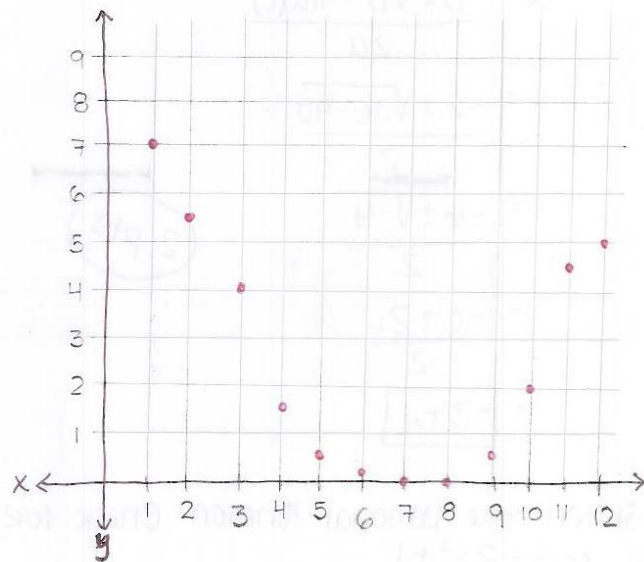
$$\begin{array}{r} 2x + \frac{1}{x} \\ x \overline{) 2x^2 + 0x + 1} \\ \underline{2x^2} \phantom{+ 1} \\ 0x + 1 \\ \phantom{0x + 1} \downarrow \\ \phantom{0x + 1} +1 \end{array}$$

4 pts



⑩ Graph the data and determine which would be the best model.

Month, $x$	Precipitation, $y$
Jan	7
Feb	5.5
Mar	4
Apr	1.5
May	0.5
June	0.2
July	0
Aug	0
Sept.	0.5
Oct	2
Nov	4.5
Dec	5



- a. linear model
- ☒ b. quadratic model
- c. ~~none~~ neither

2 pts



# Ch. 3 Test

pts total  
21

Name: KEY

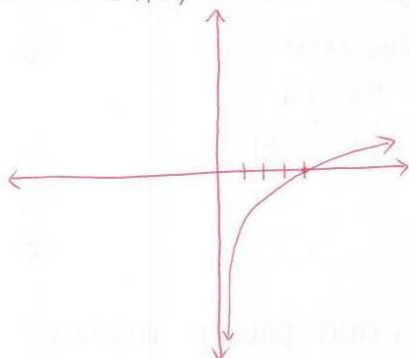
- ① Find and state the domain, vertical asymptote, & x-intercept. Sketch the graph.

$$f(x) = \log_{10}\left(\frac{x}{4}\right) \quad 4 \text{ pts}$$

Domain:  $(0, \infty)$

VA:  $x = 0$

x-int:  $(4, 0)$



- ③ Expand as a combination of multiple logarithms.

$$\ln\left(\frac{x^4 \sqrt{y}}{z^5}\right) \quad 2 \text{ pts}$$

$$\ln x^4 + \ln \sqrt{y} - \ln z^5 =$$

$$\boxed{4 \ln x + \frac{1}{2} \ln y - 5 \ln z}$$

- ② Change to base e (ln).  
 $\log_8 47$

$$\boxed{\frac{\ln 47}{\ln 8}} \quad 1 \text{ pt}$$

- ④ Condense to a single logarithm.

$$\ln x - 2(\ln(x+2) + \ln(x-2))$$

$$= \ln x - 2(\ln(x+2)(x-2))$$

$$= \ln x - 2(\ln(x^2 - 4))$$

$$= \ln x - \ln(x^2 - 4)^2$$

$$= \boxed{\ln\left(\frac{x}{(x^2 - 4)^2}\right)} \quad 2 \text{ pts}$$



⑤ Solve for  $x$ . Round to 3 decimal places. 2pts each

①  $\log_{10} x - \log_{10} (8-5x) = 2$

$$\log_{10} \frac{x}{8-5x} = 2$$

$$100 = \frac{x}{8-5x}$$

$$100(8-5x) = x$$

$$800 - 500x = x$$

②  $\ln(2x-5) - \ln x = 1$

$$\ln \frac{2x-5}{x} = 1$$

$$e = \frac{2x-5}{x}$$

$$ex = 2x-5$$

$$x \approx -6.961$$

NO SOLUTION

③  $4^{x-5} + 21 = 30$

$$4^{x-5} = 9$$

$$\log_4 9 = x-5$$

$$x = 6.585$$

④  $\log_2 X + \log_2 5 = 6$

$$\log_2 5x = 6$$

$$5x = 64$$

$$x = 12.8$$

⑥ Find and state logarithmic, exponential, and power models for the data. Which is the best fit?

4 pts

x	y
1	148.8
2	177.8
3	209.9
4	230.3
5	309.4
6	433.2
7	499.7
8	581.9

logarithmic:  $y = 200.7 \ln x + 57.835$

exponential:  $y = 115.47(1.227)^x$

power:  $y = 119.22x^{.6703}$

• exponential is the best fit.

Name:

Period #3

9 pts  
total

# Quiz

2 pts

- ① Find the complement & supplement of the angle.  
 $84^\circ$

complement:  $6^\circ$

supplement:  $96^\circ$

- ② Find the length of the arc on a circle with a radius of 15 cm intercepted by a central angle of  $60^\circ$ .

$$S = r\theta \quad \theta = 60 \times \frac{\pi}{180} = \frac{\pi}{3}$$

$$S = 15\left(\frac{\pi}{3}\right)$$

$$S = \frac{15\pi}{3} = 5\pi \text{ cm} \approx 15.71 \text{ cm}$$

- ③ The point  $(3, -4)$  is on the terminal side of an angle in standard position. Find  $\sin\theta$ ,  $\cos\theta$ , and  $\tan\theta$ .

$$r = \sqrt{3^2 + (-4)^2}$$

$$r = \sqrt{9 + 16}$$

$$r = \sqrt{25} = 5$$

$$\sin\theta = \frac{-4}{5}$$

$$\cos\theta = \frac{3}{5}$$

$$\tan\theta = \frac{-4}{3}$$

- ④ If  $\tan\theta = \frac{-12}{5}$  and  $\sin\theta > 0$ , find  $\sin\theta$  and  $\cos\theta$ .

$$r = \sqrt{5^2 + (-12)^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$r = 13$$

$$\sin\theta = \frac{12}{13}$$

$$\cos\theta = \frac{-5}{13}$$

Name:

KEY

# Quiz

(per 3)

12/2/2011

2pts each = 8 pts total

- ① Simplify  $(\csc^2 \theta)(1 - \cos^2 \theta)$ . ② Verify that

$$= (\csc^2 \theta)(\sin^2 \theta)$$

$$= \frac{1}{\sin^2 \theta} \sin^2 \theta = 1$$

$$\frac{\csc \theta + \sec \theta}{\sin \theta + \cos \theta} = \cot \theta + \tan \theta$$

$$\frac{(\csc \theta + \sec \theta)(\sin \theta - \cos \theta)}{(\sin \theta + \cos \theta)(\sin \theta - \cos \theta)} = \cot \theta + \tan \theta$$

$$\text{Right side: } \cot \theta + \tan \theta = \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta}$$

$$= \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta}$$

$$\text{Left side: } \frac{\csc \theta + \sec \theta}{\sin \theta + \cos \theta} = \frac{\frac{1}{\sin \theta} + \frac{1}{\cos \theta}}{\sin \theta + \cos \theta}$$

$$= \frac{\frac{\sin \theta + \cos \theta}{\sin \theta \cos \theta}}{\sin \theta + \cos \theta} = \frac{1}{\sin \theta \cos \theta}$$

- ③ Find  $\sin \theta$  using an addition formula.

$$\theta = 345^\circ$$

$$\sin(345) = \sin(300 + 45)$$

$$= \sin 300 \cos 45 + \cos 300 \sin 45$$

$$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{2} - \sqrt{6}}{4}$$

- ④ Find the exact values of  $\sin 2x$  and  $\cos 2x$  using the double angle formulas.

$$\cos x = \frac{-2}{\sqrt{5}}, \quad \frac{\pi}{2} < x < \pi$$

$$\sin x = \frac{1}{\sqrt{5}}, \quad \tan x = -\frac{1}{2}$$

$$\sin 2x = 2 \sin x \cos x = 2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{-2}{\sqrt{5}}\right) = -\frac{4}{5}$$

$$\begin{aligned} \cos 2x &= 2 \cos^2 x - 1 = 2\left(\frac{-2}{\sqrt{5}}\right)^2 - 1 \\ &= 2\left(\frac{4}{5}\right) - 1 \\ &= \frac{3}{5} \end{aligned}$$

# Q003

10 Pts

12/9/2011  
6.1-6.3

Name: KEY

- ① Use the Law of Sines to solve for the remaining sides and angle. (3 Pts)

$$A = 38^\circ \quad B = 58^\circ \quad a = 12$$

$$\frac{12}{\sin 38^\circ} = \frac{b}{\sin 58^\circ} \Rightarrow b = 16.53$$

$$C = 180^\circ - 38^\circ - 58^\circ = 84^\circ$$

$$\frac{12}{\sin 38^\circ} = \frac{c}{\sin 84^\circ} \Rightarrow c = 19.38$$

$$b = 16.53, c = 19.38, C = 84^\circ$$

- ② Use the Law of Cosines to solve for the angles. (3 Pts)

$$a = 7 \quad b = 15 \quad c = 19$$

$$\cos A = \frac{15^2 + 19^2 - 7^2}{2(15)(19)} = .942, A = 19.6^\circ$$

$$\cos B = \frac{7^2 + 19^2 - 15^2}{2(7)(19)} = .695, B = 45.9^\circ$$

$$C = 180^\circ - 19.6^\circ - 45.9^\circ = 114.5^\circ$$

- ③ Find the area of the triangle with sides  $a = 15$   $b = 8$   $c = 10$  (2 Pts)

$$s = \frac{a+b+c}{2} = \frac{15+8+10}{2} = 16.5$$

$$\begin{aligned} \text{Area} &= \sqrt{16.5(16.5-15)(16.5-8)(16.5-10)} \\ &= \sqrt{1367.44} \\ &= 36.98 \text{ units}^2 \end{aligned}$$

- ④ Find a unit vector in the direction of  $\vec{v} = \langle -12, -5 \rangle$ . (2 Pts)

$$\|\vec{v}\| = \sqrt{144 + 25} = \sqrt{169} = 13$$

$$\vec{u} = \left\langle \frac{-12}{13}, \frac{-5}{13} \right\rangle$$



Period 3

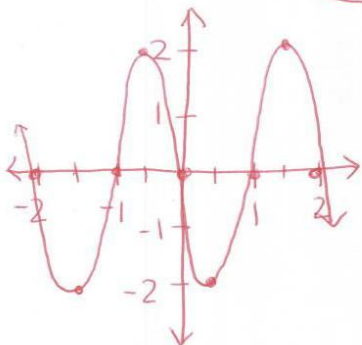
Name: KEY

# Ch. 4 Test

① Sketch the graph of the function. Include 2 full periods.

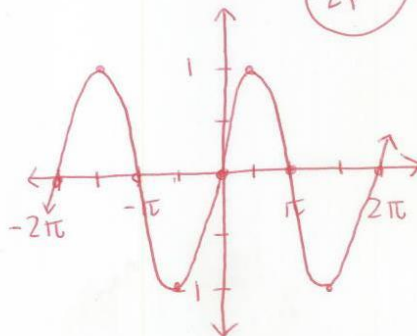
a)  $f(x) = -2\sin(\pi x)$

2pts



b)  $g(x) = \cos(x - \pi/2)$

2pts



2pts

② Find two coterminal angles, one positive and one negative, to the angle  $\frac{5\pi}{4}$ .

$\frac{13\pi}{4}, -\frac{3\pi}{4} (\pm 2\pi n)$

1pt

③ Which quadrant does  $\theta$  lie in when  $\sec\theta < 0$  and  $\tan\theta > 0$ ?

- a) Quadrant I
- b) Quadrant II
- c) Quadrant III
- d) Quadrant IV

- ④ (1 pts) What is the reference angle  $\theta'$  of the angle  $\theta = 255^\circ$ ?

$$\theta' = 255 - 180 = 75^\circ$$

- ⑤ (2 pts) If  $\sec \theta = \frac{6}{5}$  and  $\tan \theta < 0$ , find  $\sin \theta$  and  $\cos \theta$ .

$$r = 6 \quad x = 5 \quad 6^2 = 25 + y^2$$

$$\sin \theta = \frac{-\sqrt{11}}{6}$$

$$36 = 25 + y^2$$

$$y = -\sqrt{11}$$

$$\cos \theta = \frac{5}{6}$$

- ⑥ (2 pts) Use trigonometric identities to transform the left side of the equation into the right side. ( $0 < \theta < \pi/2$ )

$$\csc \theta + \tan \theta = \sec \theta$$

$$\frac{1}{\sin \theta} + \frac{\sin \theta}{\cos \theta} = \sec \theta$$

$$\frac{1}{\cos \theta} = \sec \theta$$

$$\sec \theta = \sec \theta$$

- ⑦ (2 pts) Find

a)  $\arccos\left(-\frac{\sqrt{3}}{2}\right)$

b)  $\arctan(-\sqrt{3})$

$$\frac{5\pi}{6}$$

$$-\frac{\pi}{3}$$